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# On the dimensionality of Reasoning

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Abstract: According to the categorization of reasoning tests by Kubinger (2023) in this special issue, corresponding conceptualizations of six tests were in detail also presented there. For all of them the Rasch model's validness could be fundamentally proven. And each of them seems potentially useful for practical counseling. This paper now analyzes the dimensionality of the abilities being measured with these tests. The question is if each indeed measures a specific intelligence factor but is not covered by other factors or even by a general reasoning factor – both the latter cases means that at least one of the tests is redundant and therefore is not necessarily further to discuss. Consequently, the hypothesis is: these tests are correlated only in a practical negligible extent. Because expecting a six-factor solution by applying factor analysis to six variables is hardly realistic, an analysis with multiple correlations has been tried. The question is whether the correlation coefficient between any two tests (significantly) increases if some further tests were taken into account. The pairwise correlation coefficients of eight studies were on the author's disposal (n = 2047) which all proved to be significantly non-zero, however the largest one is (only) .430, meaning a determination coefficient of 18.5% at most. When the correlation coefficient of each test as the regressand and another test as the regressor is compared with the multiple correlation coefficient using additionally one or two more tests as regressors, this always resulted in non-significance ( $\alpha = .01$ ) except on one occasion – however even there the correlation with a coefficient of .456 is anything but impressive for practice. That is, each of the tests is neither covered by another factor nor by a general reasoning factor.

Keywords: Reasoning, crystallized vs. fluid intelligence, Jäger's contents, dimensionality, multiple correlation

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# Introduction

In contrast to today's generally accepted intelligence theory "CHC" (*Cattel-Horn-Carroll*, cf. Schneider & McGrew, 2012) which focuses "reasoning" explicitly only on a *fluid* factor, "reasoning" is in need of a much broader definition from the point of view of psychological assessment in practical counseling. That is the *crystallized* facet must also be considered: "Reasoning is the ability to realize regularities and log-ically compelling connections in order to put them to appropriate use" (Kubinger, 2019, p. 244; translation by the author). Accordingly, Kubinger (2023) systemized with reference to *Adolf O. Jäger*'s three contents (*verbal* better: *lexical*, and *numerical*, *figural*) reasoning tests into six categories: Two facets times three contents.

In this special issue six respective tests were introduced. These are the *Family-Relation Reasoning-Test* (Poinstingl & Sparfeldt, 2023) representing the combination of the *crys-tallized* facet with *lexical* contents, the test *Equations* (Kubinger & Gamsjäger, 2023) as well as the test *Culture-referenced Pictographic Analogies* (Kubinger, Ünal, & Schnait, 2023), both also representing the *crystallized* facet, the former with *numerical*, the latter with *figural* contents; and there are also the tests *Reality-contradicting Syllogisms* (Treiber & Kubinger, 2023) and *Numerical Topologies* (Kubinger & Heuberger, 2023) as well as the *Two-way Figural Reasoning-Test* (Bartok & Kubinger, 2023), all three referring to the *fluid* facet, the first with *lexical*, the second with *numerical*, and the third with *figural* contents. As concerns the validness of the Rasch model all of them stood, for the time being, the test – at least after deleting a few items.

Therefore, it is now of interest, if each one of the tests indeed measures rather a specific intelligence factor, or if some of the tests are covered by group factors or even all the tests by a general reasoning factor. In the case that any of these tests is actually representable by another test as concerns the measured factor (ability dimension), then at least one of them is redundant and might not necessarily further be elaborated.

## Method

Fundamentally this is a question of factor analysis – rather one of confirmatory than one of exploratory factor analysis. However, expecting a six-factor solution applying factor analysis on only six variables (i.e. the test scores of the six tests under discussion) will hardly be realistic. Quite the opposite, confirmatory factor analysis requires at least two or three sufficiently high loading variables for each factor (cf. Kline, 2023).

Therefore, another approach has been tried. That is the analysis with multiple correlations. The question then is whether the correlation coefficient between any two tests out of the six increases (significantly) if one or two further tests were taken into account – alas, inter-correlation coefficients between groups of only four tests (apart from a single exception) were at the author's disposal, and thus higher order multiple correlation coefficients are not achievable.

The idea is to start an analysis with two tests, being of the same facet (e.g. crystallized) but with different contents (e.g. lexical and numerical). This results in an ordinary correlation coefficient (e.g.  $r_{12} = r(c)_{ln}$ ). Then the multiple correlation coefficient (of the 2<sup>nd</sup> order) is calculated, when the test with the same facet but with the third content (e.g. *figural*) is also taken into account (e.g.  $r_{1-23} = r(c)_{1-nf}$ ). Finally, the multiple correlation coefficient (of the 3<sup>rd</sup> order) is calculated, when the test with the other facet (e.g. *fluid*) and the same content (e.g. *lexical*) is additionally taken into account (e.g.  $r_{1-234} = r_{1(c)}$ n(c)f(c)l(f)). The question now is, if there are step-wise (significant) increases from  $r_{12}$  to  $r_{1-1}$ 234, in particular if a given increase from  $r_{12}$  to  $r_{1-23}$  is (significantly) larger or smaller than a given increase from  $r_{1-23}$  to  $r_{1-234}$ .<sup>1</sup> In the case that  $r_{12}$  is (significantly) larger than 0 and/or the former increase ( $r_{12}$  to  $r_{1-23}$ ) is (significantly) larger than the latter ( $r_{1-23}$  to  $r_{1-234}$ ), this fact indicates that the first test (e.g. *crystallized* facet with *lexical* contents) is rather covered by a factor "crystallized" reasoning/intelligence; on the other side in the case that  $r_{12}$  is almost 0 and the former increase ( $r_{12}$  to  $r_{1-23}$ ) is (significantly) smaller than the latter  $(r_{1-23}$  to  $r_{1-234})$ , this fact indicates that the first test (e.g. *crystallized* facet with *lexical* contents) is rather covered by a factor "lexical" reasoning/intelligence.<sup>2</sup> – Although, fundamentally our null-hypothesis is: for no test there is any significant (multiple) correlation coefficient (that is H<sub>0</sub>:  $\rho_{12} = \rho_{1-23} = \rho_{1-234} = 0$ ), we should expect some significant resulting coefficients of an however no relevant magnitude.

As the formulae of multiple correlation coefficients are not given in every textbook of statistics, they are given below. And because the formula for the multiple correlation coefficient of the third order refers to formulae of the partial correlation coefficient (of the first and the second order), these are given below as well (all the formulae with reference to Sachs, 1974).

The multiple correlation coefficient ( $2^{nd}$  order)  $r_{1-23}$  for variable 1 as the regressand and the variables 2 and 3 as regressors amounts to:

$$r_{1-23} = \sqrt{1 - (1 - r_{12}^2) \cdot (1 - r_{13,2}^2)}$$

Thereby  $r_{12}$  is the correlation coefficient for the variables 1 and 2,  $r_{13.2}$  is the partial correlation coefficient (1<sup>st</sup> order) for the variables 1 and 3 when their correlation with variable 2 is eliminated:

$$r_{13,2} = \frac{r_{13} - r_{12} \cdot r_{23}}{\sqrt{(1 - r_{12}^2)(1 - r_{23}^2)}}$$

<sup>&</sup>lt;sup>1</sup> Without loss of generality, one can assume that there are all correlation coefficients  $r_{12} \ge 0$ .

<sup>&</sup>lt;sup>2</sup> One of the reviewers correctly pointed out, that given a certain test actually does not represent a very specific factor but is covered by other factors or even a general reasoning factor and hence correlates near to 1 at least with one other test, then (higher order) multiple correlation coefficients hardly can increase; however, this case proves from the very beginning, that the test in question explicitly does not constitute a factor for its own.

 $-r_{13}$  and  $r_{23}$  being the respective correlation coefficients of variables 1 and 3 on the one hand and of variables 2 and 3 on the other hand. The multiple correlation coefficient (3<sup>rd</sup> order)  $r_{1-234}$  for variable 1 as the regressand and the variables 2, 3, and 4 as regressors amounts to:

$$r_{1-234} = \sqrt{1 - (1 - r_{12}^2) \cdot (1 - r_{13.2}^2) \cdot (1 - r_{14.23}^2)}$$

Thereby  $r_{14.23}$  is the partial correlation coefficient (2<sup>nd</sup> order) for the variables 1 and 4 when their correlation with the variables 2 and 3 is eliminated:

$$r_{14,23} = \frac{r_{14,2} - r_{13,2} \cdot r_{34,2}}{\sqrt{(1 - r_{13,2}^2)(1 - r_{34,2}^2)}}$$

Analyses were done with SPSS (Version 29), the applied syntax for the given formulae can be found in the Appendix 1.

#### Results

The data stem from the following studies<sup>3</sup> (see an overview in Table 1): Arslan (2017) with correlation coefficients of the *Family-Relation Reasoning-Test*, the test *Equations*, and the test *Culture-referenced Pictographic Analogies*; Gamsjäger (2012) with the correlation coefficient of the test *Equations* and the test *Numerical Topologies*; Grafl (2021) with correlation coefficients of the *Family-Relation Reasoning-Test*, the test *Culture-referenced Pictographic Analogies* and the test *Reality-contradicting Syllogisms*; Kresnik (2015) with the correlation coefficient of the *Family-Relation Reasoning-Test* and the test *Numerical Topologies*; Schnait (2015) with correlation coefficients of the test *Equations*, the test *Culture-referenced Pictographic Analogies*; Schnait (2015) with correlation coefficient of the *Family-Relation Reasoning-Test* and the test *Numerical Topologies*; Schnait (2015) with correlation coefficient of the *Family-Relation Reasoning-Test* and the test *Reality-contradicting Syllogisms*; Ünal (2014) with the correlation coefficient of the test *Reality-contradicting Syllogisms*; Ünal (2014) with the correlation coefficient of the test *Relation Reasoning-Test* and the test *Numerical Topologies*; and finally Winter (2016) with the correlation coefficient of the test *Culture-referenced Pictographic Analogies*, and the test *Numerical Topologies*; and finally Winter (2016) with the correlation coefficients of the test *Equations*, the test *Culture-referenced Pictographic Analogies*, and the test *Numerical Topologies*; and finally Winter (2016) with the correlation coefficients of the test *Equations*, the test *Culture-referenced Pictographic Analogies*, and the test *Numerical Topologies*; and finally Winter (2016) with the correlation coefficients of the test *Equations*, the test *Culture-referenced Pictographic Analogies*, and the *Two-way Figural Reasoning-Test*.

<sup>&</sup>lt;sup>3</sup> Each of these studies were carried out for a Master Thesis, supervised by the author of this paper as the responsible university advisor. Although all individual data for all the testees (whether an item has been solved or not) are available in the author's data archive, only the correlation coefficients as given in the theses were used here.

#### Table 1

Overview of the inter-correlation coefficients of the six reasoning-tests in question as resulted in several studies, indicated by the respective author (the sample sizes n included). Additionally two correlation coefficients are shown for the matrices test WMT 2 and one coefficient for the Figural Synthesizing Test, both tests as some substitution for the Two-way Figural Reasoning-Test. All coefficients are significant with respect to the null-hypothesis  $H_0$ :  $\rho \leq 0$ ,  $\alpha = .01$ .

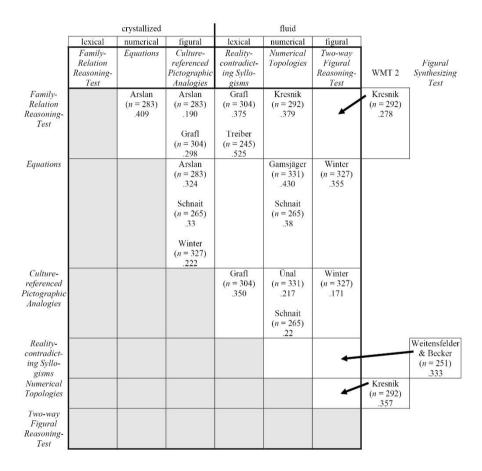


Table 1 shows that there are some pairwise combinations missing for the six tests. As the construction of the *Two-way Figural Reasoning-Test* took longer than that of the other tests, Kresnik (2015) had to use the WMT 2 (*Viennese Matrices-Test 2*; Formann, Waldherr, & Piswanger, 2011) instead. If necessary, the respective correlation coefficients were used in our analysis. Similarly, in a study by Weitensfelder and Becker (2012) the correlation coefficient of the test *Reality-contradicting Syllogisms* 

with another new *fluid-figural* reasoning-test could be used here (the task of the *Figural Synthesizing Test* is to assemble an abstract figure from several partial figures).

The formulae given above assume, of course, that all respective correlation coefficients stem from the same sample. This is not true in our case. This means that the results obtained in the following are not exact but only some approximations.

For instance for the test *Equations* [*crystallized-numerical*; n(c)] as the regressand and the *Family-Relation Reasoning-Test* [*crystallized-lexical*; 1(c)], the test *Culture-referenced Pictographic Analogies* [*crystallized-figural*; f(c)], and the test *Numerical Topologies* [*fluid-numerical*; n(f)] as regressors, the correlation coefficients result as follows (when the correlation coefficients .409, .324, .190, .430, .379, .217 from Table 1 were used):  $r_{12} = r(c)_{nl} = .4090$ ,  $r_{1-23} = r(c)_{n-lf} = .4798$ ,  $r_{1-234} = r_{n(c)-l(c)f(c)n(f)} = .5459$ the respective determination coefficients amounting to 16.73%, 23.02%, 29.81%.  $r_{12}$  $= r(c)_{ln} = .4090$  differs significantly from zero (given df = n - 2 = 200 the critical correlations coefficient is  $r_{crit} = .164$ ,  $\alpha = .01$  one-sided – see for the concerned *t*-test e.g. Rasch, Kubinger, & Yanagida, 2011<sup>4</sup>; the sample sizes are all quite larger than n= 200 which makes the critical correlation coefficient even lower). Testing the nullhypothesis  $H_0$ :  $\rho_1 = \rho_2$  with respect to the comparison of  $r_{12}$  and  $r_{1-23}$  as well as of  $r_{1-23}$ and  $r_{1-234}$  according to the pertinent (asymptotically normal distributed) test-statistic (see again e.g. Rasch, Kubinger, & Yanagida, 2011<sup>5</sup>) results in p = .296 and p = .288.

In the case where not only one, but two or three studies provide a correlation coefficient for two certain variables, the mean correlation coefficient has been calculated (see e.g. again Sachs, 1974 – the used SPSS-Syntax specifically programmed here is given in Appendix 2). That is, the given example using only the sample "Arslan", "Gamsjäger", and "Ünal" does not deliver the final result. Applying for all concerned cases the mean correlation coefficient, the respective results for each of the six tests as the regressand are shown in Table 2. Thereby, however, either  $r_{1-23}$  or  $r_{1-234}$  is rather often missing, due to the missing correlation coefficients in Table 1. For these cases at least the results concerning the comparison of  $r_{13}$  and  $r_{1-34}$  are given in Table 3, that is on one hand a certain test's correlation coefficient with a test of the same facet but of another content, and on the other hand that certain test's multiple correlation coefficient when additionally taking the test with the same content but with the other facet into account.

<sup>&</sup>lt;sup>4</sup> There also is a SPSS-Syntax given for the case of testing the null-hypothesis H<sub>0</sub>:  $\rho \ge \rho_0$ , which can be applied even for  $\rho_0 = 0$  when only a correlation coefficient is available but not the data themselves.

<sup>&</sup>lt;sup>5</sup> Again there is also given a SPSS-Syntax for that test – that Syntax has been applied here.

### Table 2

(Mean) correlation coefficients based on Table 1 and multiple correlations coefficients (2<sup>nd</sup> and 3<sup>rd</sup> order) calculated therefrom for the six reasoning-tests of interest here (sample sizes included). All correlation coefficients are significant, i.e.  $H_0: \rho = 0$  is to reject ( $\alpha = .01$ ); if the null-hypotheses  $H_0: \rho_{12} = \rho_{1-23}$  and  $H_0: \rho_{1-23} = \rho_{1-234}$  were testable the respective p-values are also given.

	<b>F</b> 12	<i>r</i> 13	<b>r</b> 23	<b>1</b> 4	<b>1</b> 24	<b>1</b> 34	<b>r</b> 1-23	<b>I*1-234</b>	H <sub>0</sub> : $p_{12} = p_{1-2}$	$H_0:$ $\rho_{1-23} = \rho_{1-1}$
Family- Relation Reasoning-	$r(c)_{\ln} = .409$		$mean r(c)_{nf} = .288$	$mean \\ r_{\rm l(c)l(f)} = .445$		$r_{\rm f(c)l(f)} = .350$	$r(c)_{l-nf} = .431$		<i>p</i> = .7571	
Test	n = 283	n = 587	n = 875	<i>n</i> = 549		<i>n</i> = 304				
Equations	$r(c)_{nl} = .409$	$mean r(c)_{nf} = .288$	$mean  r(c)_{\rm lf} = .247$	$mean  r_{n(c)n(f)} = .408$	$r_{\rm l(c)n(f)} = .379$	$mean  r_{f(c)n(f)} = .281$	$r(c)_{n-lf} = .453$	$r_{\rm n(c)-l(c)f(c)n(f)} = .514$	p = .5277	p = .3636
	n = 283	n = 875	<i>n</i> = 587	n = 596	n = 292	n = 596				
Culture- referenced Pictographic		$mean r(c)_{fn} = .288$	$mean r(c)_{ln} = .409$	$r_{\rm f(c)f(f)} = .171$	$r_{\rm l(c)f(f)} = .278$	$r_{\rm h(c)f(f)} = .355$	$r(c)_{f-ln} = .321$	$t^{r}f(c)-l(c)n(c)f(f) = .325$	<i>p</i> = .2683	p = .9557
Analogies	n = 587	n = 875	n = 283	n = 327	n = 292	n = 327				
Reality- contradict- ing Syllo-			$r(f)_{\rm H} = .333$	$mean \\ r_{l(f)l(c)} = .445$						
gisms			<i>n</i> = 251	<i>n</i> = 549						
Numerical Topologies		<i>r</i> (f) <sub>nf</sub> = .357		mean $r_{n(f)n(c)} = .408$		$r_{\rm f(f)n(c)} = .355$				
		<i>n</i> = 292		<i>n</i> = 596		<i>n</i> = 327				
Two-way Figural Reasoning-	<i>r</i> (f)fi = .333	$r(f)_{fn} = .357$		$r_{\rm f(f)f(c)} = .278$	$r_{\rm l(f)f(c)} = .350$	$mean  r_{n(f)f(c)} = .218$				
Test	n = 251	<i>n</i> = 292		n = 292	<i>n</i> = 304	n = 596				

#### Table 3

(Mean) correlation coefficients based on Table 1 and multiple correlations coefficients (2<sup>nd</sup> order) calculated therefrom for the three tests of interest here (sample sizes included). All correlation coefficients are significant, i.e.  $H_0$ :  $\rho = 0$  is to reject ( $\alpha = .01$ ). Additionally, the p-value is given as concerns the null-hypothesis  $H_0$ :  $\rho_{13} = \rho_{1.34}$ .

Family-	mean	mean	r <sub>34</sub> =	$r_{1-34} =$	H <sub>0</sub> : $\rho_{13} = \rho_{1-34}$	
Relation	$r_{13} =$	$r_{14} =$	$r_{\rm f(c)l(f)} = .350$	$r_{l(c)-f(c)l(f)} = .456$	p = .0010	
Reasoning-	$r(c)_{lf} = .247$	$r_{\rm l(c)l(f)} = .445$				
Test	1975) 17					
Numerical	$r_{13} =$	mean	$r_{34} =$	$r_{1-34} =$	H <sub>0</sub> : $\rho_{13} = \rho_{1-34}$	
Topologies	$r(f)_{nf} = .357$	$r_{14} =$	$r_{\rm f(f)n(c)} = .355$	$r_{n(f)-f(f)n(c)} = .467$	<i>p</i> = .1109	
		$r_{n(f)n(c)} = .408$			-	
Two-way	$r_{13} =$	$r_{14} =$	mean	$r_{1-34} =$	H <sub>0</sub> : $\rho_{13} = \rho_{1-34}$	
Figural	$r(f)_{\rm fn} = .357$	$r_{\rm f(f)f(c)} = .278$	$r_{\rm n(f)f(c)} = .218$	$r_{\rm f(f)-n(f)f(c)} = .412$	p = .4580	
Reasoning-	67° 95	6558 (10938)	1022.0111-1112	2000 - 2010/00/00/00		
Test						

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To summarize, all the pairwise correlation coefficients proved to be significantly nonzero, though the determination coefficient is  $(100 \cdot .430^2 =)$  18.5% at most. This indicates that the percentage of variance which mutually is explained by any two tests' relationship does not amount to even a fifth. And as concerns the quite more important questions, a) whether the third test of the same facet contributes significantly to the correlation of a facet's first and second test and b) whether the respective multiple correlation coefficient (of 2<sup>nd</sup> order) is significantly increased when the test of the other facet but the same content as the first test is additionally taken into account (i.e. multiple correlation coefficient of 3<sup>rd</sup> order), they have to be negated – with one exception: The *Family-Relation Reasoning-Test* which is of the *crystallized-lexical* type correlates to (only) .247 with the test *Culture-referenced Pictographic Analogies* (the *crystallized-figural* test), but can be significantly better predicted if additionally the test *Reality-contradicting Syllogisms* (the *fluid-lexical* test) is used (the multiple correlation coefficient of 2<sup>nd</sup> order amounts to .456).

### Discussion

Although the last-mentioned result indicates that the (lexical) content of a test is somehow predominant for a testee's achievement in comparison with the test's facet, it does not prompt any conclusion which establishes a (group) factor "lexical content". In the first instant this because that result may only be due to a type-I-error: Be aware, that such an error is not unlikely when a significance test is applied eight times (given a comparison-wise  $\alpha = .01$ , the type-I-risk amounts to .0773; cf. e.g. Rasch, Kubinger, & Yanagida, 2011); and even more important, in the second instant because the correlation coefficient's size of .456 is far from convincing (the determination coefficient amounts only to 20.8%). Within traditional factor analysis a loading of that size would never qualify a variable as a factor's marker variable. Therefore the conclusion is that the six tests of interest (and their substitutes used here) do not inter-correlate to such an extent supporting the theory of a smaller number than six factors which would explain the test scores of these tests sufficiently. Neither any test is covered by another test (or to say: factor), nor a general reasoning factor applies. Quite the opposite, each test constitutes rather a specific factor. At least for psychological counseling it seems worthwhile to represent all six reasoning categories by a respective test, so that they can then be applied specifically to the particular problem the psychologist is asked to solve.

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# **Appendix 1**

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compute r12.3=(r12-r13*r23)/sqrt((1-r13**2)*(1-r23**2)).

compute r13.2=(r13-r12*r23)/sqrt((1-r12**2)*(1-r23**2)).

compute r34.2=(r34-r23*r24)/sqrt((1-r23**2)*(1-r24**2)).

compute r14.2=(r14-r12*r24)/sqrt((1-r13.2**2)*(1-r34.2**2)).

compute r14.23=(r14.2-r13.2*r34.2)/sqrt((1-r13.2**2)*(1-r34.2**2)).

compute r1.234=sqrt((1-(1-r12**2)*(1-r13.2**2)*(1-r14.23**2))).

compute B12=r12**2.

compute B13=r13**2.

compute B1.23=r1.23**2.

compute B1.234=r1.234**2.

execute.
```

**Appendix 2** 

```
compute z01=0.5*LN((1+r01)/(1-r01)).

compute z02=0.5*LN((1+r02)/(1-r02)).

compute z03=0.5*LN((1+r03)/(1-r03)).

compute mz3=(z01*(n01-3)+z02*(n02-3)+z03*(n03-3))/(n01+n02+n03-9).

compute mz2=(z01*(n01-3)+z02*(n02-3))/(n01+n02-6).

compute mr2=(exp(2*mz2)-1)/(exp(2*mz2)+1).

compute mr3=(exp(2*mz3)-1)/(exp(2*mz3)+1).

execute.
```