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Conceptualization of the Reasoning-Test "(Mathematical) Equations"

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Abstract: Due to the six categories of reasoning tests established by Kubinger (2023a) a test concept is suggested which refers to the *crystallized* facet, concerning numerical *contents*. Each item presents a mathematical equation with two unknowns and the task is to find values for both of them which fulfill the given equation. Thereby a special multiple choice answer format is used: "2 × (1 out of 4)", that is for each unknown four options are offered. A first draft of such a test (mathematical) *Equations* with 44 items has been psychometrically analyzed according to the Rasch model. The result shows that five items have to be deleted in order to achieve *a-posteriori* model conformity. The deleted items are extensively discussed in particular with respect to the given (arrangement of the) distractors. In doing so some unintended variables could be suspected of affecting the items' difficulties: All above different strategies of processing the task seem to cause the model misfit. Consequently, some means are proposed to prevent a testee from applying the strategy of trial and error by simply checking step by step each combination of the two times four answer options until the solution is obtained.

Keywords:
Reasoning, multiple-choice response format, process strategy, distractor, Rasch model

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Introduction

Due to the classification of reasoning tests by Kubinger (2023a) there are six categories when crossing *fluid* vs. *crystallized* facets with *lexical* vs. *numerical* vs. *figural* contents. Thereby psychological assessment refers to reasoning as the "ability to realize regularities and logically compelling connections in order to put in appropriate use" (Kubinger, 2019, p. 244; translation by the authors).

In fact, hardly any test conceptualization of the category *crystallized-numerical* exist, at least none which is at practitioners' disposal provided by psychological test publishers. That fact would be justified if actually no need to measure *crystallized-numerical* reasoning arises in practice. However, we doubt that. Many occupational demands concern the ability of solving numerical problems by elementary mathematical means. That is all above every profession based on computational technology and computational science, respectively. Hence, psychological assessment regarding occupational and vocational schooling counseling with respect to the aptitude of dealing with numerical problems calls for respective tests – this is true at least until evidence proves that *crystallized-numerical* reasoning does not constitute a specific intelligence factor but is covered by some general reasoning factor.

For this an originary test concept is suggested.

Method

The test to be presented here is (mathematical) *Equations*. The task is to find the solution for both variables (*x* and *y*) of a simple mathematical equation, given some side-conditions. That is, the testee is offered four potential realizations of each variable, a single combination of which actually fulfills the equation. An item is only scored as solved if the correct combination of *x*- and *y*-options is marked (but no distractor is marked additionally).

The response format is thus a double multiple-choice format: "2 × (1 out of 4)" – "1 out of 4" meaning that a single answer option out of four is correct. This amounts to a so-called *a-priori* guessing probability (i.e. the probability to solve an item only by chance, but not with any concerning ability) of $1/4 \times 1/4 = 1/16 = .0625$. Besides the basic arithmetic operations "addition", "subtraction", "multiplication", and "division", there are also applied, for the time being, the operations (squared or cubic) "exponentiation" and "squared root". Although sometimes bracket terms as well as fraction numbers are used, all items are numerically simple, i.e. all numbers are integers and at most double-digits. Figure 1 shows the introduction item.

Example:

$x + y = 10$

x	2	8	5	10
y	9	5	1	3

The solution is $x = 5$ and $y = 5$. Only these two optional given numbers result 10 in sum.

Figure 1
The introduction item of the first draft of the test Equations: Those two values of x and y are to be found under the answer options which fulfill the equation.

The example in Figure 1 discloses that obviously the (trial and error) strategy to combine all four times four options will definitely lead to the solution. Therefore, some time restriction seems useful in order to compel the testee to instead look for a more efficient strategy of finding the solution. In the example of the given introduction item, presumably a faster strategy is to consider the x-options one after the other until the resulting y-value which consequently fulfills the equation (i.e. the sum of x and y is 10) actually exists within the given y-options. Maybe (after inspecting that all eight offered numbers are (+)10 at most) another strategy is even faster: starting from all the two numbers between 0 and 10 which sums up to 10 and then checking whether one of them is listed under the x-options, the other under the y-options.

Another example shown in Figure 2 (item 20 of parallel form A of the preliminarily test) illustrates that the first strategy, i.e. checking at worst every combination by trial and error, is in the long run hardly promising, because it is energetic-motivationally highly demanding. Instead, deliberately analyzing the equation logically seems less exhausting. That happens for instance in the following way: a) if y would be 0, then x cannot be negative but must be 0 or 2, because otherwise $2x$ never can equal x^2; as 0 is no answer option for x the solution has been found. Or b) if $x > 0$, then because of $(x^2 – y^2)$ equals $2x$ (due to reformulation) the absolute value of y must be smaller than x, however apart from $y = 0$ there is no answer option for which this applies, hence the solution is $y = 0$ and therefore $x = 2$ as already deduced above; otherwise, thinking about $x < 0$ (as just mentioned, 0 is no answer option for x), the only offered option $x = –2$ is yet not possible, because due to $y^2 = x^2 – 2x$ the value of y^2 then would be 8, the square root of which (is not an integer and) is not listed under the y-options. Finally c), given $x \neq 0$ (as the x-options determine) the case $x = y$ is impossible (again due to $y^2 = x^2 – 2x$), and hence the difference $(x^2 – y^2)$ must be even in order to get x an integer; but this only fulfills the paired numbers 0 (= y) and ±2 (= x), 0 (= y) and ±4 (= x), ±1 (either x or y) and ±3 (either y or x), ±2 (either x or y) and ±4 (either y or x) as well as for instance ±3 and ±5, ±4 and ±6, of which already the first pair ($x = 2$ and $y = 0$) turns out to be the solution (while the other pairs finally do not fulfill the equation).

20	$y^2 + 2x - x^2 = 0$	x	3	2	1	-2
		y	4	3	0	-4

Figure 2
Item 20 of parallel form B of the first draft of the test Equations (the solution is x = 2 and y = 0)

Obviously, this test conceptualization meets the facet *crystallized* and the content *numerical*. There is the need to be competent in arithmetic and algebra on one hand and to be "affable" with numbers on the other hand.

Until now a first draft for the test *Equations* has been composed with 44 items. All of them with a single solution. The offered answer options are primarily single figure numbers, sometimes double-digit numbers. The distractors have been constructed more or less arbitrarily.

As already indicated the problem with this test concept is that testees can apply different solving strategies. Hence the main psychometric effort is to prove that the items nevertheless measure the same ability, that is, they measure uni-dimensional irrespectively from the used strategy. In doing so the Rasch model is a proper means.

Thus, a study was done in order to get respective data. As all the 44 items did not seem reasonable to administer to each testee, so-called test-booklets were used. That is, the item pool of 44 items was partitioned into three non-disjunctive 20 item subpools (parallel forms), each of which administered to a group of randomly allocated testees. More precisely, the different test-booklets were arranged according to a connected incomplete block design (cf. Rasch, Kubinger, & Yanagida, 2011) in order to make parameter estimations for the item pool as a whole possible (for instance Kubinger et al., 2011, give an illustrative example of such an item linking). The design realized in this study is given in Table 1.

Table 1

The connected incomplete block design for the Equations' item pool (44 items) partitioned into three non-disjunctive 20 item sub-pools (parallel forms). Each of the 44 items is arranged in such a way that it is directly or indirectly combined with all the others (all above via the items No. 3, 10, 32, 39).

item pool No.	Item Nr.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Parallel form A	TF 1	1		2	3			4	5		6	7			8		9		10		11				12			13	
Parallel form B	TF 2		1	2		3	4			5	6			7		8			9			10		11		12			
Parallel form C	TF 3			1	2		3	4		5	6		7		8			9		10		11	12				13		14

Item pool No.	Item Nr.	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
Parallel form A	TF 1		14		15	16		17				18	19		20		
Parallel form B	TF 2	13			14		15		16		17	18		19			20
Parallel form C	TF 3			15	16		17			18		19				20	

Data sampling proceeded at Austrian police security academies with support of the federal ministry of interior. 331 trainees of the age 18 to 32 years were tested. About 80 percent were male, 20 percent female. The time limit for test execution was set to 18 minutes. As already indicated, the aim of the study was to clarify whether the Rasch model holds for the given item pool (and if not, whether *a-posteriori* model validness can be established after the deletion of very few items).

Testing the Rasch model's validness was done in accordance with state of the art (cf. Kubinger, 2005), that is Andersen's Likelihood-ratio test (LRT) was used with several partition criteria of the given overall sample into subsamples (1. score: "high-" *vs.* "low-scorers", meaning the partition in testees with a high number of solved items *vs.* testees with a low number of solved items – within each parallel form; 2. sex: male *vs.* female testees; 3. age: trainees up to 22 years *vs.* trainees older than 22 years). Given any significant LRT (comparison-wise type-I-risk $\alpha = .01$ – running three comparisons this meets a study-wise type-I-risk of approximately $\alpha = .03 < \alpha = .05$), items have been deleted step by step when repeating this model test until it resulted in non-significance for each partition criterion. The items were deleted on the basis of Rasch's graphical model check, which illustrates the coincidences of item parameter estimations when based on different subsamples: differences of any item parameter estimation between two subsamples larger than a tenth of the parameters' range indicates model misfit (cf. again Kubinger, 2005).

For analyzing the data the R-package eRm (Mair, Hatzinger & Meier, 2010) was used.

Results

Table 2 summarizes the results of Andersen's Likelihood-ratio test (LRT) with respect to the three partition criteria.

Table 2
The Rasch model tests for 44 items of the first draft of the test Equations. For the applied three criteria of partitioning the overall sample, the results for the asymptotically χ^2-distributed Andersen's Likelihood-ratio test statistic (LRT) are given as well as the degrees of freedom (df) and the respective p-value – if any item within a certain subsample is solved either by all testees or by none, that item is not included, as a consequence of which df is reduced. The results are based on 331 testees.

partition criterion	χ^2	*df*	*p*
score	64.52	38	.005
sex	50.92	43	.190
age	48.58	42	.225

Only one single significant LRT results. And this concerns the fundamentally most powerful partition criterion "high-" *vs.* "low-scorers" (cf. Kubinger, 1989). The graphical model check in Figure 3 reveals in particular a misfit of item 43. Given Rasch model's validness, each item has due to "specific objectivity" (cf. Scheiblechner, 2009) the same item (difficulty) parameter, regardless of which subsample is used; as a consequence, opposing the item parameter estimations of two subsamples in a Cartesian coordinate system would ideally result only in dots lying on a 45° line which meets the origin. However item 43's parameter estimation achieves within subsample "low-scores" a value of about 2.3 and within subsample "high-scorers" a value of about 5.0, the difference being 2.7; this is quite more than a tenth of the parameters' range (either 2.3 – (-3.0) = 5.3 or 5.0 – (-2.5) = 7.5). Beyond that, at least item 2 stands out, too.

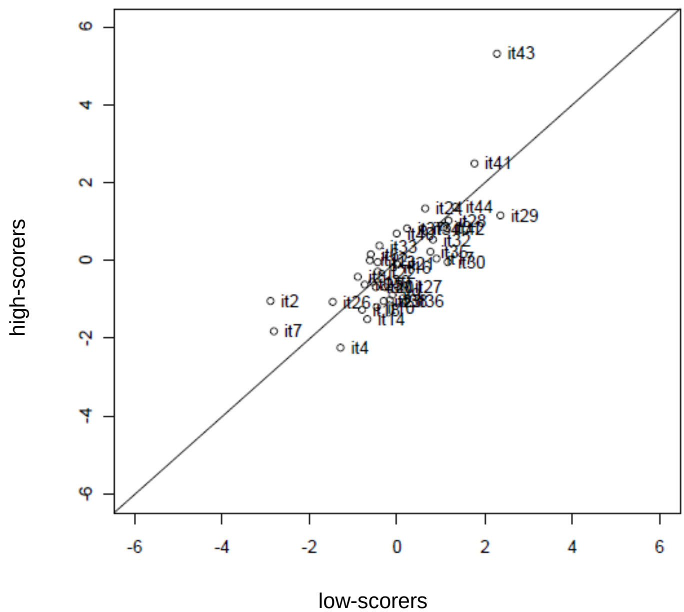

Figure 3
Graphical model check for 39 items out of the 44 item pool of the first draft of the test Equations – item (difficulty) parameter estimations according to the Rasch model as opposed for trainees with a high score (ordinate) and for trainees with a low score (abscissa)

As a matter of fact, the item difficulty parameter of item 43 is clearly larger for "high-scorers" (ordinate) than for "low-scorers" (abscissa) which means that this item is, in comparison to the other items, not as difficult for "low-scorers" as it is for "high-scorers". Looking at that item (item 20 within parallel form C) shown in Figure 4 reveals a distractor construction flaw, after all: If a testee eventually reflect upon the number 0 as the solution for x (which actually is the correct x-option), then both, $y = 2$ and $y = -2$ are equally correct options. That is, the item solution is not unequivocal. Maybe this fact upsets some "high-scorers" (more than "low-scorers", i.e. less gifted testees), discouraging them from (correctly) marking the answer options. An alternative explanation is that "high-scorers" generally use more time to analyze an item's task (and in the end finding the solution more frequently). As a consequence, "high-scorers" may get stressed out due to the given time limit, especially on the later items, and even may be urged to quit the test. Given the latter explanation holds, this should cause the test authors to reconsider the set time limits. At any rate, two-way solutions should definitely be avoided.

20	$x^2 - 2xy + y^2 = 4$	*x*	2	3	0	-1
		y	2	-2	-1	3

Figure 4
Item 43 (i.e. item 20 within parallel form C) of the first draft of the test Equations (the solution is x = 0 and y = ±2)

Actually, deleting item 43 from the pool and re-analyzing the remaining item pool led again to a significant LRT as well as the successively stepwise deletion of the items 2, 5, and 1; only after also deleting item 7, the LRT resulted in non-significance with respect to every partition criterion (see Table 3). Figures 5, 6, 7, and 8 show these four items.

Table 3
The Rasch model tests for the remaining 39 items of the first draft of the test Equations. For the applied three criteria of partitioning the overall sample the results of the asymptotically χ^2-distributed Andersen's Likelihood-ratio test statistic (LRT) are given as well as the degrees of freedom (df) and the respective p-value – if any item within a certain subsample is solved either by all testees or by none, that item is not included, as a consequence of which df is reduced. The results are based on 331 testees.

partition criterion	χ^2	*df*	*p*
score	55.17	36	.021
sex	43.43	38	.251
age	44.53	38	.216

1	$13 - x = y$	*x*	2	12	5	7
		y	6	2	9	10

Figure 5
Item 2 (i.e. item 1 within parallel form B) of the first draft of the test Equations (the solution is x = 7 and y = 6)

3	$3x = y$	x	1	10	4	7
		y	6	21	9	24

Figure 6
Item 5 (i.e. item 3 within parallel form B) of the first draft of the test Equations (the solution is x = 7 and y = 21)

1	$x = 15 – y$	x	5	12	2	7
		y	8	4	11	9

Figure 7
Item 1 (i.e. item 1 within parallel form A) of the first draft of the test Equations (the solution is x = 7 and y = 8)

4	$x : y = 4$	x	12	32	16	-4
		y	-4	-8	4	1

Figure 8
Item 7 (i.e. item 4 within parallel form A) of the first draft of the test Equations (the solution is x = 16 and y = 4)

As concerns item 2 it can be speculated that "low-scorers" re-read the equation in the sense $y = 13 – x$ as they have learned that the unknown variable is usually on the left side of an equation; then they may start by using the trial and error strategy with the first y-option $y = 6$, and consequently will easily and quickly find the solution $x = 7$. "High-scorers" however might look immediately for two numbers which sum up to 13, maybe starting from 10 + 3, 9 + 4, 8 + 5 and so on, needing perhaps more time – and thus they even abandon that (fundamentally easy) item. To counter this problem more warming-up items could be administered, giving ("high-scorers") the experience that the items' solution process lasts longer for this test than initially expected. At any rate the distractors should be arranged in such a way that the trial and error strategy does not become the fastest one for certain items. Probably the best prevention would be to remove the challenge of speed, although originally assumed to be essential. This might be achieved either by computerizing the test without any time limit or by the approach recently given by Kubinger (2021): Only the items the testee actually worked on are scored and, optionally, the completion time for a test is restricted by the time the slowest testee of the group needs until he/she has worked on a predetermined minimum number of items. However, both suggestions will only work, when the test strongly gives the impression that the trial and error strategy does not pay.

Similar as for item 2 it can be speculated for item 5 that "low-scorers" re-read the equation in the sense $y = 3x$ and, starting with the trial and error strategy, will very soon (already with the second trial: $y = 21$) find the solution $x = 7$. "High-scorers" might look immediately for two numbers with the relation 1:3, maybe starting from $x = 1$ and $y = 3$, $x = 2$ and $y = 6$, and so on, meaning a strategy which again needs more effort – and can demotivate the testee from proceeding.

As concerns item 1, "high-scorers" might immediately look, analogously to item 2, for two numbers which result in a certain sum, now 15, by possibly starting with 10 + 5, 9 + 6, and so on, which again can reduce a testee's achievement motivation from the very beginning of the test. On the other hand, "low-scorers" may use the strategy to begin with inserting a value for y, in order to calculate the respective value of x in the equation, which at the first trial will lead them to $y = 8$ and thus to the correct x-option: $x = 7$.

With respect to item 7 one can very boldly speculate that "low-scorers" exclude (somehow illogically) negative numbers under the answer options, as a consequence of which they will find the solution ($x = 16$ and $y = 4$) faster, all the more as this pairing couple of numbers is visually presented right above each other. A "high-scorer" might however be held up longer at this item.

To summarize, 5 out of 44 items have to be deleted in order to get the suggested test conceptualization so far conforming the Rasch model. That means a deletion rate of 11.4 percent; leaving item 43 out of consideration due to a distractor construction flaw, the rate lies with 9.3 percent below a tenth – the commonly tolerable rate (cf. Kubinger & Draxler, 2007).

The item parameters for the remaining 39 items lie between -2.53 and 2.38. From experience, this range falls within a medium extent. All items are shown in the Appendix.

Discussion

The conceptualization of *Equations* stood the test, for the time being. The Rasch model holds after deleting a few items. That is, a uni-dimensional measurement of some reasoning ability seems actually feasible by that conceptualization, and most probably similar items would also meet the Rasch model's validness. The deleted items do not indicate a systematic contamination of reasoning by using a certain strategy – although they disclose that the specific choice of distractors and in particular their arrangement, and the specific representation of the equation can sometimes give less gifted testees an advantage. Of course, any task similar to the ones given in the test *Equations*, essentially defining the problem of an item by the distractors, requires extreme carefulness when constructing them. At the end, the presented first draft of the suggested test concept accidentally misses the necessary carefulness.

Therefore, for the test's progress an even more deliberate construction of the distractors is highly recommended. First of all, the items for a next draft should be checked with the method of thinking out loud. This exploratory method asks a subject to work on the items while he/she verbally expresses all thought processes and action strategies. Unexpected effects, as retrospectively speculated above, causing an item's nonconformity with the Rasch model would most likely be detected soon. Of course, a statistical distractor analysis should be done additionally. If the answer frequencies of the distractors for any item do not result equally distributed, at least one distractor has to be revised or substituted: A distractor which is marked very often (perhaps as frequently as the correct answer option or even more) indicates a systematically misleading task (for some people); on the other side a distractor which is marked rarely or almost never discloses that even to less gifted testees it appears obvious to not be the solution.

As already mentioned, some additional warming-up items should be used, in particular to demonstrate that the trial and error strategy is not the method of choice but rather energetic-motivational highly demanding. An example shown in Figure 9 may serve as a further introduction item in order to encourage the testee not to apply the trial and error strategy, as for instance an alternative strategy is more efficient there: Because the result of the equation is positive, one easily realizes that no x larger than the largest given y can be correct, and of course no negative value of y does; hence, the solution must be $x = 2$ and $y = 4$.

Example II:

$$3 \cdot (y - x) = 6$$

x	6	5	2	7
y	-1	-3	-2	4

Figure 9
A suggested additional introduction item for the test Equations (the solution is x = 2 and y = 4)

Furthermore, it seems worthwhile to consider the use of more than four answer options for each variable. This would most likely also help to prevent a testee from applying the trial and error strategy. However alternatively, analyzing the equation logically would hardly increase the energetic-motivational demands.

As repeatedly indicated, the use of time limits should be reconsidered. When the test is computerized, an item-wise time limitation is feasible which might reduce the stress (for a "high-scorer") to reach the last items. Given the suggested interventions actually help to prevent testees from applying the trial and error strategy, a computerized test would not need any time limit at all; and for paper and pencil testing in groups just setting a predetermined termination according to the slowest processing testee as mentioned above would work.

Finally, we recommend constructing items by using some item generating rules, that is the underlying logical structure of the equation and answer options is scheduled in advance by some appropriate cognitive operation components (i.e. "radicals"). This allows (all above by means of the so-called LLTM – linear logistic test model; Fischer, 1973; see also Fischer, 2005, as well as Kubinger, 2008, 2009) to test statistically, whether these rules and cognitive operations components, respectively, do sufficiently explain the items' difficulties and therefore definitely determines a test's validity as the ability to apply these cognitive operations components properly. Furthermore, if they actually sufficiently explain the items' difficulties, one may compose new items by combining some of the components in such a way that an item difficulty results which is looked for – this is especially advantageous for adaptive testing (cf. Kubinger, 2016). For instance, Bartok and Kubinger (2023) as well as Kubinger and Heuberger (2023) illustrate this approach. If, however, these cognitive operation components fail to explain the items' difficulty, then this proves that the items' difficulties are affected by unintended variables. In particular, different strategies of processing the task, as discussed above, must then considered as one such variable.

If *crystallized-numerical* reasoning as aimed to be measured by the test *Equations* constitutes indeed rather a specific intelligence factor than is covered by other factors or even covered by a general reasoning factor, has for now been analyzed by Kubinger (2023b).

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Appendix

The Rasch model conform 39 items of the first draft of the test Equations (the correct answer options printed in bold)

Item						
3	$y + 5 = x$	x	7	**3**	4	12
		y	3	6	**-2**	-5
4	$x = 4y$	x	32	12	8	**28**
		y	4	**7**	10	1
6	$x \cdot y = 30$	x	2	-6	**-3**	4
		y	-15	12	**-10**	5
8	$2(x - y) = 6$	x	6	**5**	0	1
		y	-8	6	7	**2**
9	$3(y - x) = 9$	x	-8	**2**	6	7
		y	6	0	1	**5**
10	$(3x + 2y - 2) = 5$	x	0	**1**	2	3
		y	**2**	5	3	4
11	$\frac{24}{x} + y = 8$	x	6	1	**4**	8
		y	7	**2**	-4	0
12	$\frac{18}{x} + y = 6$	x	18	6	**3**	1
		y	2	**0**	-3	4
13	$\frac{24}{x} = 8 - y$	x	1	24	**8**	3
		y	9	**5**	6	-4
14	$x^2 = y + 13$	x	3	-1	7	**-5**
		y	13	10	**12**	-11
15	$x + 15 = y^2$	x	2	0	3	**1**
		y	-1	6	**-4**	2
16	$x^2 = 2y$	x	1	2	**-6**	-8
		y	8	50	1	**18**
17	$x^2 = 4y$	x	12	2	**-6**	-8
		y	25	-16	-1	**9**
18	$x^2 : y = 4$	x	-6	**-4**	3	1
		y	1	-2	**4**	2
19	$x^2 : y = 2$	x	-6	**-2**	1	3
		y	1	-1	**2**	8
20	$\frac{y}{x^2} = 4$	x	-4	0	**-2**	1
		y	36	**16**	5	8
21	$3 = \frac{x}{y^2}$	x	-3	9	**12**	27
		y	1	**-2**	-4	0
22	$x^2 = 22 + 2y$	x	14	10	1	**6**
		y	**7**	61	-9	21
23	$y^2 = 3x + 33$	x	37	-8	16	**1**
		y	**6**	1	18	15
24	$(x - y)^2 = 4$	x	4	**2**	3	1
		y	5	-5	4	**0**
25	$(x - y)^2 = 0$	x	1	**3**	6	0
		y	4	2	5	**3**
26	$(x + y)^2 = 25$	x	6	**2**	8	-3
		y	1	0	-5	**3**

27	$2x - x^2 = y$	*x*	2	4	**1**	-2
		y	2	3	-4	**1**
28	$x = 3y - y^2$	*x*	24	9	**2**	-3
		y	3	-1	4	**1**
29	$x = 2y^2 - 5y$	*x*	-7	18	**-2**	-3
		y	3	-1	4	**2**
30	$\frac{x^2 - 4x}{2} = y$	*x*	4	12	1	**6**
		y	16	30	1	**6**
31	$\frac{8x - x^2}{4} = y$	*x*	-4	8	12	**4**
		y	-20	1	12	**4**
32	$\frac{x + y}{x - y} = 2$	*x*	2	-6	**3**	4
		y	**1**	0	-1	-5
33	$\sqrt{x} = 2y$	*x*	16	**4**	1	9
		y	3	9	0	**1**
34	$3x = \sqrt{y}$	*x*	9	**1**	0	3
		y	4	36	1	**9**
35	$2x^2 = \sqrt{y}$	*x*	2	3	-2	**1**
		y	7	**4**	1	9
36	$\sqrt{x} = 3y^2$	*x*	7	4	1	**9**
		y	2	**1**	-2	3
37	$3^x + y = 11$	*x*	1	3	**2**	0
		y	**2**	0	1	3
38	$2^y + x = 13$	*x*	3	0	**5**	1
		y	**3**	2	1	0
39	$x + (x - y)^2 = 0$	*x*	4	5	3	**0**
		y	3	**0**	2	5
40	$x^3 + y^2 = 2$	*x*	0	**1**	-1	-2
		y	**-1**	2	0	-3
41	$y^3 + x^2 = 1$	*x*	-1	**3**	0	-2
		y	**-2**	0	2	-1
42	$y^2 + 2x - x^2 = 0$	*x*	3	**2**	1	-2
		y	4	3	**0**	-4
44	$x + 2xy + y = 7$	*x*	1	0	**-2**	-1
		y	0	-1	1	**-3**