

Modeling and predicting non-linear changes in educational trajectories: The multilevel latent growth components approach

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Abstract

The investigation of developmental trajectories is a central goal of educational science. However, modeling and predicting complex trajectories in the context of large-scale panel studies poses multiple challenges. Statistical models oftentimes need to take into account a) potentially non-linear shapes of trajectories, b) multiple levels of analysis (e.g., individual level, university level) and c) measurement models for the typically unobservable latent constructs. In this paper, we develop a new approach, termed the *multilevel latent growth components model* (ML-LGCoM) that can adequately address all three challenges simultaneously. A key feature of this new approach is that it allows researchers to test contrasts of interest among latent variables in a multilevel study. In our illustrative example, we used data from the National Educational Panel Study to model the (non-linear) development of students' satisfaction with their academic success over four years while taking into account cluster- and individual-level trajectories and measurement error.

Keywords: multilevel structural equation modeling, latent growth components, longitudinal models, latent state trait, educational trajectories

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Introduction

The investigation of developmental trajectories is a central goal of educational science. Researchers are interested how a person's competencies, attitudes, and motivation change, and develop over a (long) period of time. Large-scale panel studies like the National Educational Panel Study (NEPS) in Germany are designed to provide representative, longitudinal data about educational processes. However, the statistical analysis of specific research questions about changes in an outcome-of-interest is not straightforward using data from large-scale studies. For example, investigating a research question like "Does a change of major after the first year of studies improve students' satisfaction with their academic success sustainably?" requires a statistical model that considers three major challenges: a) the potentially non-linear trajectories of satisfaction, b) the multiple levels of analysis (i.e., the individual level, the university level etc.), and c) a measurement model as satisfaction is a non-observable, latent variable (i.e., to account for measurement error).

There exists a wide range of different methods to analyze longitudinal data. Proposed methods include latent growth curve models (McArdle & Epstein, 1987; M. Meredith & Tisak, 1990), hierarchical linear models (HLM, Goldstein, 2003), latent change score models (McArdle & Hamagami, 2001; Raykov, 1999; Steyer, Eid, & Schwenkmezger, 1997), latent growth component models (Mayer, Steyer, & Mueller, 2012), and autoregressive models with or without cross-lagged effects (Hertzog & Nesselroade, 2003; Jöreskog, 1979). McArdle (2009) provides an overview of some of these models. However, few of them have been extended to explicitly deal with all the challenges of large-scale panel studies identified above. Important steps in this direction are the extension of growth curve models to include measurement models (multiple-indicator LGCMs, McArdle, 1988; Tisak & Tisak, 2000) and flexible shapes of trajectories (LGCMs with estimated loadings, McArdle & Epstein, 1987; piecewise LGCMs, Bollen & Curran, 2006), the extension of linear growth curve models to multilevel designs (Muthén, 1997), and the extension of HLMs to handle three and more levels. We discuss the use of the latter two models for the analysis of trajectories in large-scale panel studies.

Multilevel models for trajectories in educational panel studies

Multilevel latent growth curve models (ML-LGCM; Muthén, 1997) can be used to model linear, quadratic or higher order polynomial change processes in multilevel designs. These models allow for modeling individual as well as cluster-specific trajectories in latent variables. The observed variables (e.g., satisfaction with academic success, SAS) are decomposed into cluster-level variables (e.g., SAS at the university level) and individual-level variables (e.g., individual deviations from university level SAS). Then, linear trajectories of SAS are modeled with an intercept and a slope variable at both the university level and the individual level. ML-LGCMs require researchers to impose a prespecified functional form for all individual trajectories. This is, within ML-LGCMs the trajectory of a students' satisfaction with his academic success has to be assumed

as, for example, linear or quadratic. For many dynamic outcomes of interest such as SAS the linearity assumption is inappropriate, as a linear trajectory of satisfaction does not allow to examine dynamic reactions to recent or critical events. For example, when students change major, university or their level of academic degree (from Bachelor to Master), it is very likely that individual trajectories have other functional forms or even are discontinuous.

Hierarchical linear models for three (or more) level data (HLM, Goldstein, 2003) are widely used in the context of educational studies. These models have been developed to account for complex multilevel structures and can handle two or more levels. In our example, HLMs need to be formulated as three-level models with time points nested in individuals nested in universities, while the multivariate ML-LGCM formulation only requires two levels (individual and university). HLMs can also be used to model non-linear changes but they do not include measurement models for the latent variables. That is, HLMs assume that there is no measurement error in the manifest variables used to measure the latent constructs of interest. This assumption of perfectly reliable measures for the outcomes is too strong in the context of educational trajectories and can lead to biased point estimates and standard errors of the statistical model.

Nowadays, the boundaries between the SEM and the HLM framework have gradually disappeared. For both frameworks there exist generalizations to include the strength of the respective other, this is modern multilevel structural equation modeling (ML-SEM, B. O. Muthén & Asparouhov, 2008) and the generalized linear latent and mixed modeling (GLLAMM, Skrondal & Rabe-Hesketh, 2004). As a consequence, most models can be formulated in both frameworks (for more details see Mehta & Neale, 2005 or Rabe-Hesketh, Skrondal, & Zheng, 2012).

Latent growth component models

In this paper, we propose a comprehensive statistical model adequately addressing all three challenges simultaneously and modeling the trajectory of an outcome-of-interest in its full complexity. For this purpose, we build on an alternative to various kinds of growth curve models for multilevel designs, namely the so-called latent growth component model (LGCoM, Mayer et al., 2012). While the LGCoM has not yet been extended to multilevel data, it has some key advantages for modeling educational trajectories compared to other approaches: The LGCoM can be used to model non-linear contrasts of change and also include adequate measurement models for latent variables. Measurement models are an important feature in modeling educational trajectories, because constructs such as SAS are not directly observable and are therefore measured by multiple indicators. The relationship between these indicators and the latent variable of interest is then specified in the measurement model. LGCoMs form the basis for the new approach that will be developed in this paper.

To date, the shortcoming of classic LGCoMs is that they do not account for the complex multilevel structure frequently encountered in educational panel studies, i.e., the

non-linear contrasts are only applied at the individual level and not at the cluster level. Ignoring the cluster level in education panel studies not only gives limited information for policy makers but also leads to deflated standard errors. Taking into account the multilevel structure in the analysis is important for both substantive and statistical reasons. From a substantive point of view, it allows researchers to examine educational trajectories on different levels simultaneously, for example, how students' individual satisfaction develops during their studies and how the average satisfaction develops on an institutional level. Educational trajectories can be quite different at the individual level and at the group/institution level. Ignoring such differences limits the information that we get from our studies. In multilevel models, researchers can not only model the trajectories on different levels but can also include covariates that predict educational trajectories at different levels, i.e., individual-level covariates, institution-level covariates and contextual covariates. Contextual covariates (sometimes also referred to as compositional variables, for example, Hutchison, 2007) reflect the composition of cluster-level units, for example, the average achievement level in an institution. The importance of considering effects of such contextual covariates has long been discussed under the keyword contextual effects (e.g., Raudenbush & Willms, 1995).

From a statistical point of view, ignoring the cluster level in education panel studies violates the assumption of independently sampled observations that many statistical models require. In educational studies, observations are typically not independent, because students from the same cluster (e.g., class or university) are more similar than students from different clusters, and ignoring this fact would lead to deflated standard errors of effect estimates. If studying the individual level only, a correction of the standard errors and model fit measures may suffice (Stapleton, 2008). Multilevel modeling is required if effects are examined at both the cluster- and the individual level. Another statistical issue that arises when the multilevel structure is not adequately taken into account is bias in effect estimates. The cluster variable itself and functions thereof like contextual covariates can be confounding variables that need to be considered to obtain correct estimates (Mayer, Nagengast, Fletcher, & Steyer, 2014).

We will use the multilevel structural equation modeling framework (ML-SEM, McDonald & Goldstein, 1989) to extend the latent growth components approach (Mayer et al., 2012) to multilevel designs. The new approach, a multilevel latent growth components model (ML-LGCoM), will then be illustrated with an example from the NEPS datasets, where we model trajectories of students' satisfaction with their academic success (SAS) both at the individual level and at the study program level. All models will be specified using the lavaan package (Rosseel, 2012). The code is provided in the supplemental materials.

Multilevel latent growth components

In this section, we will explain step-by-step the constituting parts of a multilevel latent growth components model (ML-LGCoM). We start with the multistate model (Steyer,

Mayer, Geiser, & Cole, 2014) and show how growth components can be defined and interpreted within the single-level LGCoM. In the last step, the expansion to the multilevel framework is presented.

The multistate model

In longitudinal panel studies, we are oftentimes interested in trajectories of latent constructs. The constructs are measured at multiple time points and we use η_t to denote a latent construct, such as students' satisfaction with their academic success, measured by multiple indicators Y_{it} , $i = 1, \dots, I$ at time point t , $t = 1, \dots, T$. In latent state-trait theory (LST theory, Steyer, Ferring, & Schmitt, 1992; Steyer et al., 2014), which forms the theoretical foundation for our work, the η_t variables are also called latent state variables, since they represent attributes of a person-in-a-situation. Multiple indicators per time point are required to be able to distinguish measurement error influences and true scores. A LST model that allows for partitioning the variance of the observed indicators into measurement error variance and true score variance in longitudinal studies is the multistate model. This model is the basis for the single-level latent growth component model. We use a multistate model with a time-invariant measurement model (W. Meredith, 1993; Widaman & Reise, 1997) resulting in the following model equations for a specific time point t :

$$\begin{aligned} Y_{1t} &= 0 + 1 \cdot \eta_t + \epsilon_{1t} \\ Y_{2t} &= \lambda_{20} + \lambda_{21}\eta_t + \epsilon_{2t} \\ Y_{3t} &= \lambda_{30} + \lambda_{31}\eta_t + \epsilon_{3t} \\ &\vdots \\ Y_{It} &= \lambda_{I0} + \lambda_{I1}\eta_t + \epsilon_{It}. \end{aligned}$$

In matrix notation, the multistate model is a special case of the measurement model used in SEM:

$$\mathbf{y} = \boldsymbol{\nu} + \mathbf{\Lambda}\boldsymbol{\eta} + \boldsymbol{\epsilon} \quad (1)$$

where \mathbf{y} denotes the vector of observed variables, $\boldsymbol{\nu}$ is the vector of time-invariant intercepts λ_{i0} , $\mathbf{\Lambda}$ denotes a matrix of time-invariant loadings λ_{i1} , and $\boldsymbol{\eta}$ and $\boldsymbol{\epsilon}$ denote the vectors of latent state and measurement error variables, respectively. The multistate model is one of the models that can be estimated within LST theory. It can be extended to include traits (i.e., stable attributes of a person) or method factors (i.e., indicator-specific components not shared with other indicators) if necessary. For an overview of different ways to handle method effects see Geiser and Lockhart (2012).

Single-level latent growth component model

In the multistate model, the latent state variables η_t are allowed to covary and their means are estimated without restrictions, but no trajectories or changes between them are

modeled. The latent growth components model (LGCoM, Mayer et al., 2012) is a very flexible approach that allows for the specification of non-linear contrasts and trajectories of latent state variables. Thus, it can be used to test substantive hypotheses about changes in educational trajectories. Relations or contrasts among the state variables are specified and represented by so-called *growth component variables*. In the LGCoM the relations among several η_t can easily be defined as contrasts. Contrasts basically transform the latent state variables η_t into growth component variables π_m , which then represent the substantive hypotheses about non-linear changes in trajectories. For example, in a study with four measurement occasions (and hence, four latent state variables) four possible growth components are given by:

$$\begin{aligned}\pi_0 &= -1 \cdot \eta_1 + 0 \cdot \eta_2 + 0 \cdot \eta_3 + 1 \cdot \eta_4 \\ \pi_1 &= -3 \cdot \eta_1 + (-1) \cdot \eta_2 + 1 \cdot \eta_3 + 3 \cdot \eta_4 \\ \pi_2 &= -3 \cdot \eta_1 + 1 \cdot \eta_2 + 1 \cdot \eta_3 + 1 \cdot \eta_4 \\ \pi_3 &= 0 \cdot \eta_1 + 0 \cdot \eta_2 + 0 \cdot \eta_3 + 1 \cdot \eta_4\end{aligned}$$

The coefficients of this contrast equations can be comprised into the contrast matrix

$$\mathbf{C} = \begin{pmatrix} -1 & 0 & 0 & 1 \\ -3 & -1 & 1 & 3 \\ -3 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

The first row represents a latent difference score between η_1 and η_4 . The second row represents a linear growth component. In the third row, we compare η_1 to the mean of η_2 , η_3 and η_4 . This kind of growth component can be useful to model a sustainable change after a critical life-event (for an example see Mayer, Geiser, Infurna, & Fiege, 2013). The fourth row represents η_4 , i.e., the fourth growth component is identical to the fourth state variable. Note that this procedure allows for a multitude of non-linear contrasts among the latent state variables with or without a given functional form. More formally, the vector of growth component variables $\boldsymbol{\pi}$ is defined as a function of the vector of latent state variables $\boldsymbol{\eta}$:

$$\boldsymbol{\pi} = \mathbf{C}\boldsymbol{\eta} \quad (3)$$

This implies that the latent state variables can be computed as a function of the growth components,

$$\boldsymbol{\eta} = \mathbf{C}^{-1}\boldsymbol{\pi} \quad (4)$$

which we can use to extend our multistate model to a latent growth component model:

$$\mathbf{y} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta} + \boldsymbol{\epsilon} \quad (5)$$

$$\boldsymbol{\eta} = \mathbf{C}^{-1}\boldsymbol{\pi}, \quad (6)$$

The π_m variables are a complete decomposition of the η_t variables, i.e., the additional structural model (Equation 6) is saturated. Hence, the model fit of the multistate model and the latent growth component model are identical. Note that the LGCoM can also be written in a single equation by inserting Equation 6 into Equation 5: $\mathbf{y} = \boldsymbol{\nu} + \boldsymbol{\Lambda}(\mathbf{C}^{-1}\boldsymbol{\pi}) + \boldsymbol{\epsilon}$.

Multilevel latent growth components model

The LGCoM accounts for measurement error and non-linear trajectories of latent state variables – two of the three previously described challenges in analyzing non-linear trajectories in large-scale educational studies. In this paper, we extend the LGCoM to multilevel designs and propose the multilevel latent growth components model (ML-LGCoM). Especially educational researchers often have to deal with nested data structures, for example, students nested in classes, classes nested in schools, students in universities and so forth.

We denote the cluster variable by C^1 and will use the multilevel SEM framework for our full ML-LGCoM. In this framework, the growth component variables, the latent state variables and the manifest indicators are decomposed into within and between components. For the growth component variables, this decomposition is given by:

$$\boldsymbol{\pi} = \boldsymbol{\pi}_b + \boldsymbol{\pi}_w, \quad (7)$$

where $\boldsymbol{\pi}_b = E(\boldsymbol{\pi}|C)$ and $\boldsymbol{\pi}_w = \boldsymbol{\pi} - E(\boldsymbol{\pi}|C)$. For example, consider a simple growth component π_0 defined as the difference between the latent state variable satisfaction at the first and at the second occasion: $\pi_0 = \eta_2 - \eta_1$. In the multilevel extension, π_0 is decomposed into a between level component π_{0b} , whose values reflect the change in cluster-level satisfaction between the two time points, and π_{0w} , whose values reflect individual deviations from the cluster-level changes.

The extension of the LGCoM to a multilevel framework is based on a decomposition of the manifest observed variables \mathbf{Y} into a cluster- or between-level \mathbf{Y}_b and an individual- or within-level part \mathbf{Y}_w , with a respective latent growth components model on each level:

$$\begin{aligned} \mathbf{Y}_w &= \boldsymbol{\Lambda}_w \boldsymbol{\eta}_w + \boldsymbol{\epsilon}_w \\ \boldsymbol{\eta}_w &= \mathbf{C}^{-1} \boldsymbol{\pi}_w \\ \mathbf{Y}_b &= \boldsymbol{\nu} + \boldsymbol{\Lambda}_b \boldsymbol{\eta}_b + \boldsymbol{\epsilon}_b \\ \boldsymbol{\eta}_b &= \mathbf{C}^{-1} \boldsymbol{\pi}_b. \end{aligned}$$

The between-part \mathbf{Y}_b is defined as conditional expectation of \mathbf{Y} given a cluster variable C , while the within-part is considered as residual. Thus, the expectation of the within-part is

¹To be in line with previous research, we use the italicized C for the cluster variable and the bold faced \mathbf{C} for the contrast matrix in the LGCoM.

zero by definition, i.e., $E(Y_w) = 0$. The decomposition can generally be modeled in two ways - either using the observed cluster means as estimator for the between-part (Enders & Tofighi, 2007; Kreft, de Leeuw, & Aiken, 1995; Raudenbush & Bryk, 2002) or using a latent approach correcting for sampling error in the observed variables (Lüdtke et al., 2008). Throughout this article, we will apply the latent aggregation approach.

Predictors of growth components

Typically, researchers are not only interested in describing patterns of change but also in predictors that may explain certain aspects of individual or cluster-level differences in change. Within the ML-SEM framework, predictors $\mathbf{Z} = (Z_1, Z_2, \dots, Z_k)'$ of growth components $\boldsymbol{\pi}$ can be included using linear regressions,

$$E(\pi_w | \mathbf{Z}_w) = \boldsymbol{\beta}'_w \mathbf{Z}_w \quad (8)$$

$$E(\pi_b | \mathbf{Z}_b) = \beta_0 + \boldsymbol{\beta}'_b \mathbf{Z}_b, \quad (9)$$

where \mathbf{Z}_w denotes the within-part and \mathbf{Z}_b denotes the between-part of the covariates \mathbf{Z} , and $\boldsymbol{\beta}_w$ and $\boldsymbol{\beta}_b$ denote the vectors of regression coefficients on each level respectively. This extends the multilevel latent growth components model to

$$\mathbf{Y}_w = \boldsymbol{\Lambda}_w \boldsymbol{\eta}_w + \boldsymbol{\epsilon}_w$$

$$\boldsymbol{\eta}_w = \mathbf{C}^{-1} \boldsymbol{\pi}_w$$

$$\boldsymbol{\pi}_w = \boldsymbol{\beta}'_w \mathbf{Z}_w + \boldsymbol{v}_w$$

$$\mathbf{Y}_b = \boldsymbol{\nu} + \boldsymbol{\Lambda}_b \boldsymbol{\eta}_b + \boldsymbol{\epsilon}_b$$

$$\boldsymbol{\eta}_b = \mathbf{C}^{-1} \boldsymbol{\pi}_b$$

$$\boldsymbol{\pi}_b = \beta_0 + \boldsymbol{\beta}'_b \mathbf{Z}_b + \boldsymbol{v}_b.$$

where v_b and v_w denote the regression's residuum on the between- and within-level.

Empirical example: Students' satisfaction

We use NEPS data to illustrate how a ML-LGCoM can be specified in the ML-SEM framework and applied to answer substantive research questions in large-scale educational studies. In our illustrative example, we focus on the core elements of ML-LGCoM and additionally consider predictors of growth components on the cluster- as well as on the individual level. We model a) the development of university students' satisfaction with their academic success (SAS) over four years, b) cluster- and individual-level trajectories of SAS, c) measurement error in the items measuring SAS, and d) predictors of trajectories on different levels.

Substantive background

Study satisfaction The latent construct of students' satisfaction with their academic studies can be considered as one criterion of overall study success, which is a multidimensional construct comprising both objective and subjective indicators (Kuh, Kinzie, Buckley, Bridges, & Hayek, 2007). Students' satisfaction has been found to be related to, for instance, academic achievement (Wach, Karbach, Ruffing, Brünken, & Spinath, 2016) and retention (Schneider & Nelson, 2013). In the university context, students' satisfaction with their academic studies can refer to different aspects (e.g., study contents, organization of the study program, teaching quality). However, Wach et al. (2016) point out that there is a lack of a commonly accepted definition of students' satisfaction with their academic studies. In this paper, we will refer to study satisfaction as students' satisfaction with their own performance as it appears closely related to academic goal achievement. We will use a part of the "fulfillment of achievement expectations" scale (Trautwein et al., 2007) to measure it².

Trajectories of study satisfaction The transition from school to university can be a challenging, yet exciting experiences for new students. In an analogy to what has been called the "Honeymoon-Hangover Effect" (Boswell & Boudreau, 2005) in organizational job-change research, one might expect the initial semesters are accompanied by an increase in study satisfaction immediately following the transition (honeymoon effect), eventually followed by some decline, thereafter (hangover effect). Alternatively, one might refer to the general hedonic treadmill model of well-being adaptation processes (Brickman & Campbell, 1971; Frederick & Loewenstein, 1999), which suggests that the reaction to a positive (negative) event increases (decreases) well-being but that then, after an adaptation phase, well-being returns to previous levels.

So far, there is little knowledge on the trajectories of study satisfaction of university students. A study by Hiemisch, Westermann, and Michael (2005) with students of medicine and dentistry showed their satisfaction to decline from the beginning of their first term to their second term. Hiemisch et al. (2005) discuss this effect as reflecting the elevated mood of students who are admitted to medical school, which is difficult to achieve. This enthusiasm might then be attenuated as coping with the high demands of the study program gets to the fore. Their study, however, did solely comprise two measurement points, which makes it impossible to analyze (and speak of) trajectories (see also studies by Wach et al., 2016; Singley, Lent, & Sheu, 2010). A study with three measurement points was published by Schmitt, Oswald, Friede, Imus, and Merritt (2008): On average, students showed a decline of academic satisfaction from the end of the first semester to the end of the second semester, but no change from the end of the second to the end of the third semester. Most importantly, based on latent growth models, Schmitt et al.

²In NEPS, there is also a nine-item scale (Westermann, Heise, Spies, & Trautwein, 1996) on student satisfaction. However, this scale was only administered at two waves. Therefore it cannot be used to model longitudinal trajectories.

(2008) demonstrated that changes in students' perceived fit between interest/skills and the study program were positively associated with changes in academic satisfaction. Recently, Upadaya and Salmela-Aro (2017) analyzed data from a five-wave longitudinal study with two measurements during post-comprehensive education and three measurements at university or vocational school. They used a questionnaire of work/study engagement focusing on aspects such as dedication to study/work and enthusiasm about studies/work, which can be seen as indicating study satisfaction. They showed, on average, increases in satisfaction during the transition from post-comprehensive school to university/work. In addition, they found cross-lagged effects from performance to this kind of domain-specific satisfaction (as well as to general life satisfaction). A recent study by Janke, Rudert, Marksteiner, and Dickhäuser (2017) showed declining satisfaction with studying over time. However, the authors point out that their samples are not representative for the German higher education system, as they were limited to students of one institution or study program. In sum, partly in contrast to theoretical assumptions such as the "honeymoon-hangover effect" and the "hedonic treadmill", previous studies revealed evidence for a declining pattern of satisfaction, though there is also some support for stabilizing trends.

Variability of SAS trajectories There are strong differences between study programs with respect to basic characteristics of learning environments. Commonly, four dimensions are used to distinguish the quality of a learning environment: structure, support, challenge, and orientation (Bäumer, Preis, Roßbach, Stecher, & Klieme, 2011). Challenge, for instance, refers to the cognitive demands of a learning environment (e.g., difficulty of examinations). As study programs differ in these dimensions, it is unlikely that SAS trajectories are identical across all study programs. Instead, it is plausible to assume that there are differences in trajectories depending on the chosen study program.

Not only the study program contributes to heterogeneity in SAS trajectories, but there are also interindividual differences in SAS trajectories. For some students, the first major may have turned out to be dissatisfying – for other students the first major may rise hopes and curiosity with respect to future social and academic integration. So the valence of the transitional process is not unequivocal. Although the idea of well-being only fluctuating around a biologically determined set point has been challenged, empirical research has shown that some adaptation does occur but with considerable differences in how close and fast individuals return to former well-being levels (e.g., Diener, Lucas, & Scollon, 2006).

Effects of change of major The lack of knowledge on the long-term development of students' satisfaction is even more evident with respect to the satisfaction trajectories of those who quit or change majors. Clearly, not all students acquire degrees in the study programs they had chosen. In Germany, the dropout from Bachelor programs at universities is estimated to be about 33 percent (Heublein, Richter, Schmelzer, & Sommer,

2014). Though, reliable official estimates of major changes are surprisingly rare. For university students, a change of major represents a serious non-normative transition. From a content point of view, we approach the topic of changing major by asking whether and how this change affects student well-being in terms of young people's temporal pattern of study satisfaction. As students who change their major will most likely do so to optimize the (so far low) fit between interest/abilities and the contents/requirements of a study program, this should have beneficial performance effects, including being satisfied with this performance.

Effects of burden of examination According to previous studies (Nauta, 2007; Apenburg, 1980; Wach et al., 2016), the (perceived) difficulty (in terms of academic achievements) of a study program is strongly related to students' satisfaction with their academic success. However, one would expect differential effects between our levels of analysis. On the one hand, a study program characterized by a high examination burden, raises the probability of dissatisfying events among students, for example, poor or failed exams. This might lead to lowered satisfaction in demanding study programs. On the other hand, individuals might perceive high demands of their study program (e.g., examination burden) as motivating and, therefore, increase their efforts to succeed (Wach et al., 2016). This effect has been described using the framework of goal-setting theory (Locke & Latham, 2002). With respect to well-being of working adults, it has been shown that progress in career goals led to well-being increase solely if perceived goal difficulty had been high (Wiese & Freund, 2005). Accordingly, a specific high goal is stated to have a positive linear relationship with task performance and to affect one's satisfaction, as it serves as evaluative standard. Vice versa, a student underestimating the demands of his study program is likely to put less effort into learning, possibly resulting in lower grades.

Research questions

In this paper, we will use the newly developed multilevel latent growth components approach to model and predict trajectories of students' satisfaction with their academic success. Specifically, we examine if the trajectories follow, on average, a non-linear form, characterized by an initial increase in satisfaction, followed by a return to the initial level of satisfaction. And we test whether the size and direction of the non-linear changes varies between study programs and between individuals. Finally, we will look at the effects of a change of major and examination burden on the trajectories. In concrete terms, we hypothesize that the more burdensome the examinations of a study program are perceived, the more the average levels of students' satisfaction with their academic success will decrease over time. Based on goal-setting theory, we expect to find the opposite effect at the student level, i.e., we expect a positive relationship between individual perceptions of the examination burden and the development of individual satisfaction with academic success. With respect to students who change their major after the first year of studies,

we expect lower satisfaction levels in the first year compared to the remaining years of studies.

Method

Sample

To illustrate the ML-LGCoM, we use data from the National Educational Panel Study (NEPS) study in Germany (Blossfeld, Roßbach, & von Maurice, 2011). NEPS is a multi-cohort longitudinal study aimed at examining lifelong educational processes. For our example, we use data from the Starting Cohort 5 (first-year students). Within the first-year students cohort, students' satisfaction with their academic success was assessed four times, each about one year apart (i.e., after the first, second, third and fourth year of studies). The initial sample size in this four waves was $N_1 = 14610$ and dropped to $N_4 = 8629$ in the fourth wave. We used listwise deletion for the analysis. The available sample size for the complete model with all predictors is $N = 2054$. In this article, we do not present a comprehensive analysis of the NEPS first-year students cohort. Instead, the primary goal of our paper is to illustrate the ML-LGCoM approach as a means to model and predict non-linear trajectories in educational processes. For didactic purposes, we restrict ourselves to students' satisfaction as outcome, to an exploratory set of non-linear contrasts, and to a limited set of predictors (perceived examination burden and change of major).

Measures

Students' satisfaction with their academic success The scale "students' satisfaction with their academic success" consists of three items from the "fulfillment of achievement expectations" scale (Trautwein et al., 2007):

- Y_{1t} : My academic achievements (grades) are better than I had originally expected.
- Y_{2t} : I have fully met my own expectations for my performance and grades in these studies.
- Y_{3t} : I am satisfied with my performance in the studies.

The items are coded on a Likert scale ranging from 1 = *does not apply at all* to 4 = *applies completely* and t denotes the measurement occasion with $t = 1, \dots, 4$. We used these three positively worded items as indicators of a common latent state variable η_t using a η -congeneric measurement model. Note that all individuals are first assessed in their first semester, this is, the time variable t represents time since the start of the study program. The time variable at the institution level corresponds to the individual level. Therefore the time variable t on both levels always corresponds to a specific wave of the NEPS study. The reliability of the satisfaction with academic success scale was McDonald's $\omega_H = .81$.

Study programs For the first-years students cohort in NEPS, clusters of all students enrolled in a certain field of study at a particular higher education institution were drawn. For example, all students studying social sciences at the (public) University of Bamberg in the first wave form one cluster. In total, study programs are summarized into 60 different fields of study, for example, architecture, mathematics, psychology, educational science and so forth. Within each cluster, all students are surveyed. In our analyses we used this cluster variable C as level-2 units representing study programs in Germany. Note, that this variable only captures the study program at the first wave and does not account for possible changes of study programs in the following years.

Predictors Every year, students were asked if they had changed their major since the last survey. We used this dichotomous item Z_1 to explain interindividual variability in change of satisfaction after the first year of studies. Additionally, an item asking to what extend the study program is characterized by a high examination burden (coded from 1 = *very little* to 5 = *a lot*) was used to explain differential trajectories of change among study programs (Z_{2b}) and individuals (Z_{2w}).

Statistical models

We directly use a multilevel multistate model as starting point for our analysis (skipping the single-level models in our presentation of the statistical models). Next, we specify and add the latent growth components and, finally, the predictors of growth components.

Multilevel multistate model

We first formulated a multilevel multistate model of students' satisfaction with their academic success using lavaan 0.6-1.1183 (Rosseel, 2012; code provided in Appendix C). The latent state variables η were specified with a time- and level-invariant measurement model. To deal with correlated measurement error variables across time points between the three SAS indicators, we had to add method factors to the multilevel multistate model as suggested by Pohl, Steyer, and Kraus (2008). The within and between measurement model for the multilevel multistate model with method factors are shown in Figure 1. The complete model equations are shown in Appendix A.

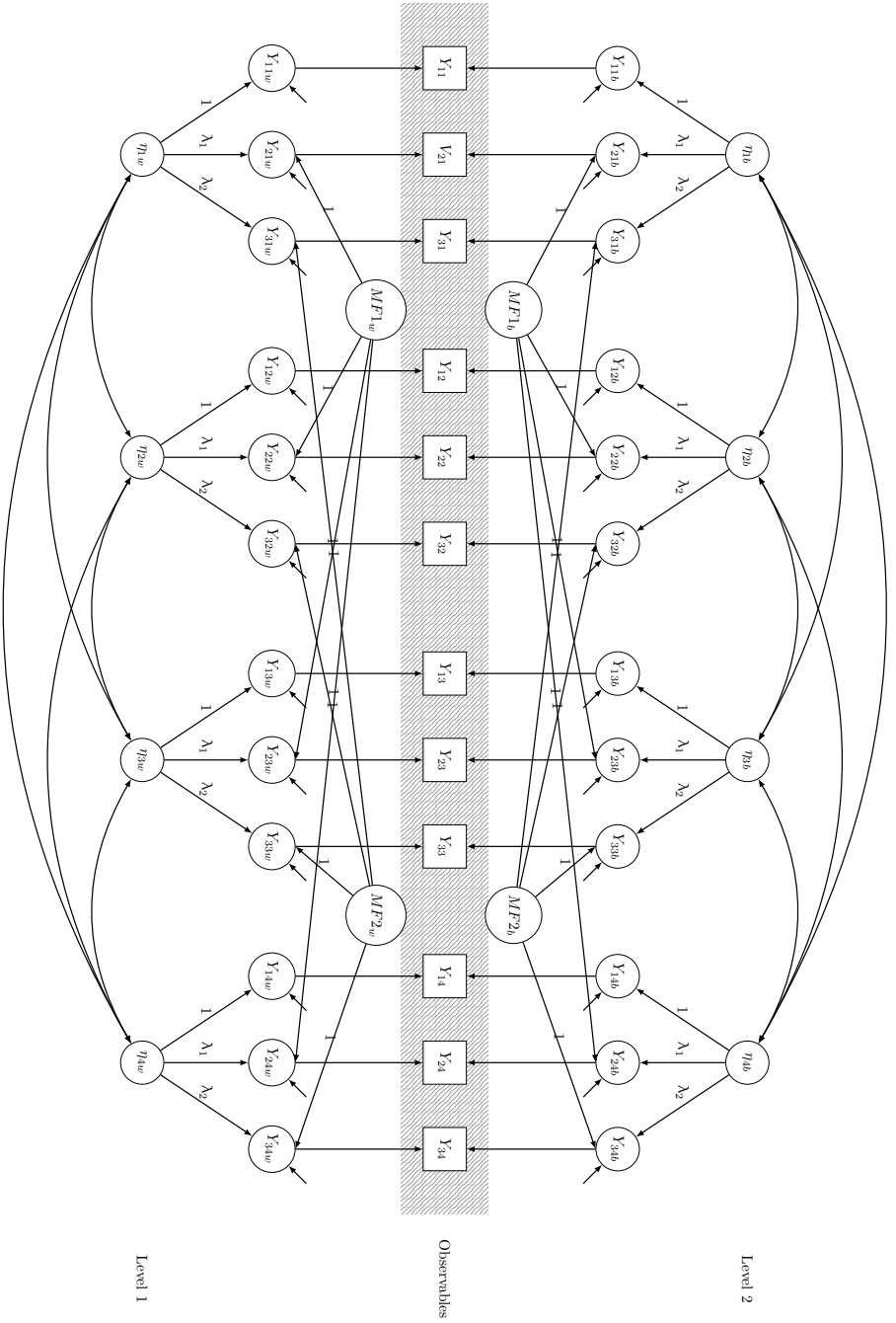


Figure 1:

Path diagram for the multilevel multistate model with method factors.

Multilevel growth components

Next, we specified a contrast matrix \mathbf{C} to define the latent growth components according to our research questions:

$$\begin{aligned}\pi_{0w} &= 1 \cdot \eta_{1w} \\ \pi_{1w} &= -3 \cdot \eta_{1w} + 1 \cdot \eta_{2w} + 1 \cdot \eta_{3w} + 1 \cdot \eta_{4w} \\ \pi_{2w} &= -2 \cdot \eta_{2w} + 1 \cdot \eta_{3w} + 1 \cdot \eta_{4w} \\ \pi_{3w} &= -1 \cdot \eta_{3w} + 1 \cdot \eta_{4w},\end{aligned}$$

and equivalently for the between part:

$$\begin{aligned}\pi_{0b} &= 1 \cdot \eta_{1b} \\ \pi_{1b} &= -3 \cdot \eta_{1b} + 1 \cdot \eta_{2b} + 1 \cdot \eta_{3b} + 1 \cdot \eta_{4b} \\ \pi_{2b} &= -2 \cdot \eta_{2b} + 1 \cdot \eta_{3b} + 1 \cdot \eta_{4b} \\ \pi_{3b} &= -1 \cdot \eta_{3b} + 1 \cdot \eta_{4b},\end{aligned}$$

resulting in the contrast matrix:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

The values of the first growth component *initial level* π_0 are the true scores of students' satisfaction after the first year of studies (i.e., the values on η_1). This variable serves as an intercept variable that reflects the initial true level of students' satisfaction. The latent variable *overall change after first year* π_1 represents a contrast between the true satisfaction scores of the first occasion and the average of the true scores of the following three years. A negative score on this growth component would mean that the satisfaction level in the years 2 to 4 was overall lower than in the first year of studies. This growth component therefore reflects a sustainable change, averaging across annual fluctuations. Accordingly, the growth components *overall change after second* π_2 and *third year* π_3 reflect the overall change in students' satisfaction levels after the second and the third year of studies. These growth components examine (sustainable) changes in later stages of studies, when adaption to the study program has already taken place.

As described above, the inverse of the contrast matrix \mathbf{C} is needed in the structural model to specify the latent growth components models.

$$\mathbf{C}^{-1} = \begin{pmatrix} -\frac{1}{3} & -\frac{1}{6} & -\frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The resulting equations for the within and between structural parts of the model are:

$$\begin{pmatrix} \eta_{1w} \\ \eta_{2w} \\ \eta_{3w} \\ \eta_{4w} \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & -\frac{1}{6} & -\frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \pi_{0w} \\ \pi_{1w} \\ \pi_{2w} \\ \pi_{3w} \end{pmatrix}$$

$$\begin{pmatrix} \eta_{1b} \\ \eta_{2b} \\ \eta_{3b} \\ \eta_{4b} \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & -\frac{1}{6} & -\frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \pi_{0b} \\ \pi_{1b} \\ \pi_{2b} \\ \pi_{3b} \end{pmatrix}$$

Finally, we included the covariates Z_1 (change of major), Z_{2b} (average perceived examination burden), and Z_{2w} (individual perceived examination burden) to examine their influence on trajectories of students' satisfaction. The growth components were regressed on the covariates on their respective level. A path diagram for the full model is shown in Figure 2.

Results

Multilevel multistate model

First, we fitted a multilevel multistate model with time- and level-invariant measurement models and two method factors using the maximum likelihood (ML) estimator in lavaan. The multilevel multistate model showed an adequate fit $\chi^2(106, N = 2076) = 233.566$, $RMSEA = .024$, $CFI = .989$, $SRMR_w = .017$, $SRMR_b = .033$. Detailed results for this model are shown in Table 1. Descriptively, the means of the latent state variables went up and down over time, ranging from $M(\eta_1) = 2.295$ to $M(\eta_2) = 2.416$. The inter-cluster differences in the latent satisfaction scores were highest at the first and the fourth time point, while interindividual differences dropped between the first and second year and remained stable then, which is reflected by the variances of the latent state variables.

Multilevel latent growth components

In the next step, we added the growth components to the multistate model with method factors to obtain more detailed information about changes in satisfaction across time. Since the growth components represent a saturated decomposition of the latent state variables, the growth component model and the multistate model with method factor have the same model fit. The estimated means and variances of the growth components are shown in Table 2. Note that the mean and variance of the growth components π_{0w} and π_{0b} are identical with the corresponding parameters of the latent state variables η_{1b} and η_{1w} in the multistate model with method factors, as we defined the growth components that way.

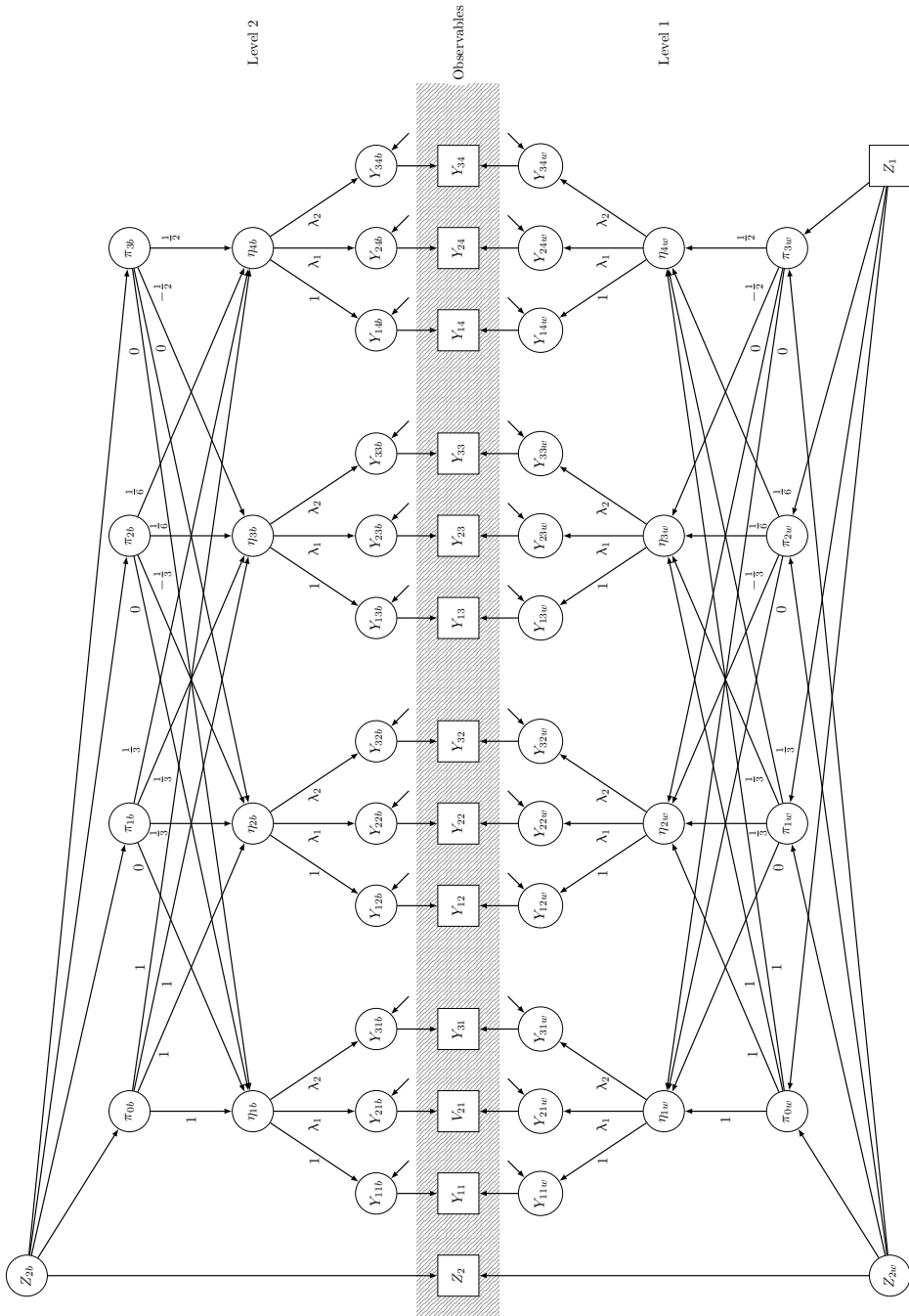


Figure 2: Path diagram for the multilevel latent growth components model (method factors and covariances between latent variables are not shown).

Table 1:
Results for the between-level multistate model with method factors

Between-Level					
	Estimate	SE		Estimate	SE
Means			Residual		
			Variances		
$M(\eta_{1b})$	2.295	0.024	$Var(\epsilon_{11b})$	0.000	
$M(\eta_{2b})$	2.416	0.021	$Var(\epsilon_{21b})$	0.000	
$M(\eta_{3b})$	2.383	0.023	$Var(\epsilon_{31b})$	0.000	
$M(\eta_{4b})$	2.409	0.023	$Var(\epsilon_{12b})$	0.000	
$M(MF_{1b})$	0.376	0.085	$Var(\epsilon_{22b})$	0.000	
$M(MF_{2b})$	-0.110	0.112	$Var(\epsilon_{32b})$	0.000	
Variances			$Var(\epsilon_{13b})$	0.000	
$Var(\eta_{1b})$	0.042	0.011	$Var(\epsilon_{23b})$	0.000	
$Var(\eta_{2b})$	0.028	0.008	$Var(\epsilon_{33b})$	0.000	
$Var(\eta_{3b})$	0.040	0.011	$Var(\epsilon_{14b})$	0.000	
$Var(\eta_{4b})$	0.044	0.011	$Var(\epsilon_{24b})$	0.000	
$Var(MF_{1b})$	0.003	0.002	$Var(\epsilon_{34b})$	0.000	
$Var(MF_{2b})$	0.012	0.005			
Within-Level					
	Estimate	SE		Estimate	SE
Variances			Residual		
			Variances		
$Var(\eta_{1w})$	0.402	0.017	$Var(\epsilon_{11w})$	0.192	0.010
$Var(\eta_{2w})$	0.344	0.014	$Var(\epsilon_{21w})$	0.214	0.010
$Var(\eta_{3w})$	0.369	0.016	$Var(\epsilon_{31w})$	0.150	0.011
$Var(\eta_{4w})$	0.364	0.015	$Var(\epsilon_{12w})$	0.204	0.009
$Var(MF_{1w})$	0.144	0.011	$Var(\epsilon_{22w})$	0.201	0.009
$Var(MF_{2w})$	0.214	0.017	$Var(\epsilon_{32w})$	0.143	0.009
			$Var(\epsilon_{13w})$	0.189	0.009
			$Var(\epsilon_{23w})$	0.173	0.008
			$Var(\epsilon_{33w})$	0.141	0.008
			$Var(\epsilon_{14w})$	0.196	0.009
			$Var(\epsilon_{24w})$	0.185	0.008
			$Var(\epsilon_{34w})$	0.156	0.010

Note: Model fit: $\chi^2(106, N = 2076) = 233.566$, RMSEA = .024, CFI = .989, SRMR_w = .017, SRMR_b = .033. Covariances between all latent variables were included but are not shown. The measurement error variances at the between level $Var(\epsilon_{itb})$ have been set to 0.0001.

The mean of the initial between-level growth component π_{0b} reflected the average level of students' satisfaction after the first year, $M(\pi_{0b}) = 2.295$ ($SE = 0.024$). The mean of

Table 2:
Means and variances of latent growth components

	Estimate	SE		Estimate	SE
$M(\pi_{0b})$	2.295	0.024	$Var(\pi_{0b})$	0.042	0.011
$M(\pi_{1b})$	0.323	0.043	$Var(\pi_{1b})$	0.045	0.025
$M(\pi_{2b})$	-0.039	0.025	$Var(\pi_{2b})$	0.018	0.009
$M(\pi_{3b})$	0.025	0.012	$Var(\pi_{3b})$	0.001	-
			$Var(\pi_{0w})$	0.402	0.017
			$Var(\pi_{1w})$	1.574	0.103
			$Var(\pi_{2w})$	0.476	0.035
			$Var(\pi_{3w})$	0.109	0.008

Note: Model fit: $\chi^2(106, N = 2076) = 233.565$, RMSEA = .024, CFI = .989, SRMR_w = .017, SRMR_b = .033

the between-level growth component π_{1b} was $M(\pi_{1b}) = 0.323$ ($SE = 0.043$) indicating an increase of average satisfaction scores on the cluster-level. In order to calculate the average increase of satisfaction, we have to divide 0.323 by three ($0.323/3 = 0.108$). This is because the growth component π_{1b} has been defined in such a way that the sum of the satisfaction scores of the last three occasions is compared to three times the score at the initial occasion. Accordingly, the means of the other two growth components $M(\pi_{2b}) = -0.039$ ($SE = 0.025$) and $M(\pi_{3b}) = 0.025$ ($SE = 0.012$) indicated that average satisfaction did hardly change after the second year on the between-level.

The variance of the initial between-level growth component was $Var(\pi_{0b}) = 0.042$ indicating differences in the initial satisfaction levels among study programs. The variances of the first between-level growth component $Var(\pi_{1b}) = 0.045$ reflected some noticeable differences of average satisfaction change between clusters. However, these differences decrease over time, as is indicated by $Var(\pi_{2b}) = 0.018$ and $Var(\pi_{3b}) = 0.001$.

Interindividual differences in change of satisfaction after the first year were quite large $Var(\pi_{1w}) = 1.574$, while the interindividual variability of change after the second and third year decreased, i.e., $Var(\pi_{2w}) = 0.476$ and $Var(\pi_{3w}) = 0.109$.

Effects of predictors on growth components

We included predictors in the complete model by simultaneously estimating the regression of the growth components on the predictors change of major Z_1 , between-level examination burden Z_{2b} and within-level examination burden Z_{2w} . Table 3 shows the results for regression analyses. The model fit for the complete model is $\chi^2(146) = 479.735$, RMSEA = 0.033, CFI = .972, SRMR_b = 0.059, SRMR_w = 0.043.

Students' who changed their major after the first year of studies, were significantly less satisfied with their academic success than those who continued with their major

($\beta_{10w} = -0.240, p < .001$). However, the change of major predicted a sustainable gain in satisfaction ($\beta_{11w} = 0.695, p < .001$) after the first year, while later changes were not significantly predicted by the change of major after the first year.

On the between-level, there was no significant relation between the perceived burden of examination of a study program and the average level of students' satisfaction after the first year ($\beta_{20b} = -0.127, p = .099$). However, burden of examination was negatively linked to change of students' satisfaction ($\beta_{21b} = -0.502, p = .002$), indicating that the decrease of satisfaction was higher in study programs high in perceived burden of examination. This effect was still significant for changes after the second year ($\beta_{22b} = -0.297, p = .001$). In contrast, on the individual level, perceived burden of examination was negatively related to students' satisfaction at first ($\beta_{20w} = -0.133, p < .001$), but the sustainable changes after the first ($\beta_{21b} = 0.267, p < .001$) and second year ($\beta_{22b} = 0.061, p = .038$) were both positively related to this perception.

Table 3:
Regressions with predictors

	Between-level							
	$E(\pi_{0b} Z_{2b})$		$E(\pi_{1b} Z_{2b})$		$E(\pi_{2b} Z_{2b})$		$E(\pi_{3b} Z_{2b})$	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Burden of examination	-0.127	0.078	-0.502	0.164	-0.297	0.095	0.015	0.047
	Within-level							
	$E(\pi_{0w} Z_1, Z_{2w})$		$E(\pi_{1w} Z_1, Z_{2w})$		$E(\pi_{2w} Z_1, Z_{2w})$		$E(\pi_{3w} Z_1, Z_{2w})$	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Burden of examination	-0.133	0.019	0.267	0.048	0.061	0.029	0.006	0.015
Change of major	-0.240	0.063	0.695	0.157	-0.124	0.096	0.020	0.049

Note: Model fit: $\chi^2(146) = 479.735, CFI = 0.972, RMSEA = 0.033, SRMR_b = 0.059, SRMR_w = 0.043$

Discussion

In this article, we showed how to define latent growth components in a multilevel structural equation modeling framework. The decomposition of the manifest indicators, the latent state variables, and the latent growth component variables into within and between components in the newly developed ML-LGCoM allows for considering contrasts of interest at different levels. It makes it possible, for example, to model the development of the cluster-level averages of a level 1 construct and the development of the individual deviations from the cluster-level averages separately. This is particularly interesting when there are considerable differences between the two levels. In addition, covariates at both levels can be used to predict the within and between parts of the latent growth components.

In our empirical example, we used data from four waves of the National Educational Panel Study to look at changes in students' satisfaction with their academic success both at the study program level and on the level of the individual student. Three different growth components were constructed to contrast latent state variable at the first, second,

and third wave with the subsequent wave(s). At the study program level, we found that latent satisfaction increases significantly after the first wave and remains relatively constant afterwards on average. There is variation in this pattern both between study programs and between individuals. A significant predictor was change of major after the first year, which predicted a sustainable gain in satisfaction. At the study program level, burden of examination of the study program was negatively linked to change of students' satisfaction, indicating that the decrease of satisfaction was higher in study programs high in perceived burden of examination.

Further developments of the ML-SEM framework will likely yield additional options for the application and development of multilevel latent growth components models. In particular, accounting for cross-classified multilevel structures in multilevel structural equation models would be a major improvement. By cross-classified structure, we refer to a multilevel structure, which is not strictly hierarchical (e.g., when students change from one study program to another; for details and two examples of a cross-classified structure, see Raudenbush, 1993). In addition, the consideration of interindividual time heterogeneity and thus, different meanings of time on the institutional and the individual level (e.g., a student in his third semester at university, but the first semester in a new study program) would be insightful (for details on modeling of individual time points see Blozis & Cho, 2008).

Throughout our paper, we always examined the between- and within-part of a single contrast variable π . In general, the model also allows for examining different contrasts on each level, which might be useful if the meaning of time differs among levels. Conceptually, this would imply the use of two contrast matrices C_1 with growth components $\pi = \pi_b + \pi_w$, and C_2 with growth components $\zeta = \zeta_b + \zeta_w$. Researchers could then choose to study π_w or ζ_w on the within-level and π_b or ζ_b on the between-level. Thus, there would be a slight change in notation to clarify that between- and within-parts of two distinctly defined growth components are inspected.

The ML-LGCoM is a very flexible model with broad applications in large-scale educational studies and other studies with a multilevel design. In the future, the growth components constructed within the ML-LGCoM can also be used as predictors of an outcome-of-interest. This would open up new options to examine the long term effects of a specific change pattern. For example, our approach can be used to investigate whether students who show a steep early increase in study satisfaction also have better grades at the end of their curriculum or finish earlier compared to students with a less steep increase. The ML-LGCoM currently is designed for modeling contrasts between latent state variables, which contain both trait-specific aspects and situational fluctuations. In designs with more occasions of measurement, the ML-LGCoM could also be extended to model contrasts between latent trait variables, in a similar manner as the approach presented by Eid and Hoffmann (1998).

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A. Model equations for the multistate model

$$\begin{pmatrix} Y_{11w} \\ Y_{21w} \\ Y_{31w} \\ \vdots \\ Y_{34w} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & 0 & 0 & 0 & 1 & 0 \\ \lambda_2 & 0 & 0 & 0 & 0 & 1 \\ \vdots & & & & & \\ 0 & 0 & 0 & \lambda_2 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_{1w} \\ \eta_{2w} \\ \eta_{3w} \\ \eta_{4w} \\ MF_{1w} \\ MF_{2w} \end{pmatrix} + \begin{pmatrix} \varepsilon_{11w} \\ \varepsilon_{21w} \\ \varepsilon_{31w} \\ \vdots \\ \varepsilon_{34w} \end{pmatrix}$$

and

$$\begin{pmatrix} Y_{11b} \\ Y_{21b} \\ Y_{31b} \\ \vdots \\ Y_{34b} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & 0 & 0 & 0 & 1 & 0 \\ \lambda_2 & 0 & 0 & 0 & 0 & 1 \\ \vdots & & & & & \\ 0 & 0 & 0 & \lambda_2 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_{1b} \\ \eta_{2b} \\ \eta_{3b} \\ \eta_{4b} \\ MF_{1b} \\ MF_{2b} \end{pmatrix} + \begin{pmatrix} \varepsilon_{11b} \\ \varepsilon_{21b} \\ \varepsilon_{31b} \\ \vdots \\ \varepsilon_{34b} \end{pmatrix}$$

B. From contrast matrix to code

For our illustrative example, we used the contrast matrix

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

to define the latent growth components.

The inverse contrast matrix is

$$\mathbf{C}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 1 & \frac{1}{3} & \frac{1}{6} & -\frac{1}{2} \\ 1 & \frac{1}{3} & \frac{1}{6} & \frac{1}{2} \end{pmatrix}$$

and represents how the latent state variables are computed from the latent growth components, this is

$$\begin{aligned} \eta_1 &= 1 \cdot \pi_0 + 0 \cdot \pi_1 + 0 \cdot \pi_2 + 0 \cdot \pi_3 \\ \eta_2 &= 1 \cdot \pi_0 + \frac{1}{3} \cdot \pi_1 + -\frac{1}{3} \cdot \pi_2 + 0 \cdot \pi_3 \\ \eta_3 &= 1 \cdot \pi_0 + \frac{1}{3} \cdot \pi_1 + \frac{1}{6} \cdot \pi_2 + -\frac{1}{2} \cdot \pi_3 \\ \eta_4 &= 1 \cdot \pi_0 + \frac{1}{3} \cdot \pi_1 + \frac{1}{6} \cdot \pi_2 + \frac{1}{2} \cdot \pi_3 \end{aligned}$$

Hence, the first column gives the coefficients of π_0 measured by the latent state variables η_t . This yields the resulting code in lavaan (e.g., for the within level):

```

1 pi1w =~ (1)*eta1w + (1)*eta2w + (1)*eta3w + (1)*eta4w
2 pi2w =~ (0)*eta1w + (.33333)*eta2w + (.33333)*eta3w + (.33333)*eta4w
3 pi3w =~ (0)*eta1w + (-.33333)*eta2w + (.16667)*eta3w + (.16667)*eta4w
4 pi4w =~ (0)*eta1w + (0)*eta2w + (-.5)*eta3w + (.5)*eta4w

```

The first column of C^{-1} yields the coefficients for the first row of code, the second column for the second row and so on.

C. lavaan input

```

1 # Model Specification on the Within-Level
2 level : 1
3
4 # Definition of the Latent State Variables eta
5 ## Loadings la2 and la3 are time-invariant
6 eta1w =~ 1*tg53211_2 + la2*tg53212_2 + la3*tg53213_2
7 eta2w =~ 1*tg53211_4 + la2*tg53212_4 + la3*tg53213_4
8 eta3w =~ 1*tg53211_6 + la2*tg53212_6 + la3*tg53213_6
9 eta4w =~ 1*tg53211_8 + la2*tg53212_8 + la3*tg53213_8
10
11 # Inclusion of two method factors to deal with correlated uniqueness across time
    points
12 mf1w =~ 1*tg53212_2 + 1*tg53212_4 + 1*tg53212_6 + 1*tg53212_8
13 mf2w =~ 1*tg53213_2 + 1*tg53213_4 + 1*tg53213_6 + 1*tg53213_8
14
15 # Variances of the Latent State Variables are Fixed to Zero
16 # Because the Variables are Completely Decomposed into the Growth Components
17 eta1w ~ 0*eta1w; eta2w ~ 0*eta2w; eta3w ~ 0*eta3w; eta4w ~ 0*eta4w
18
19 # Growth Components Defined via the Latent State Variables
20 # Using the Inverse Contrast Matrix C
21 # For More Details How to get from C to these coefficients see Appendix B
22 pi1w =~ (1)*eta1w + (1)*eta2w + (1)*eta3w + (1)*eta4w
23 pi2w =~ (0)*eta1w + (.33333)*eta2w + (.33333)*eta3w + (.33333)*eta4w
24 pi3w =~ (0)*eta1w + (-.33333)*eta2w + (.16667)*eta3w + (.16667)*eta4w
25 pi4w =~ (0)*eta1w + (0)*eta2w + (-.5)*eta3w + (.5)*eta4w
26
27 # Regressions of Growth Components on the Predictors
28 # tg51300_4 is the change of major before the second wave
29 # t245403_2 is the perceived burden of examination
30 pi1w ~ tg51300_4 + t245403_2
31 pi2w ~ tg51300_4 + t245403_2
32 pi3w ~ tg51300_4 + t245403_2
33 pi4w ~ tg51300_4 + t245403_2

```

```

34
35
36 # Model Specification on the Between-Level
37 level : 2
38
39 # Definition of the Latent State Variables eta
40 ## Loadings la2 and la3 are time- (and also level-) invariant
41 eta1b =~ 1*tg53211_2 + la2*tg53212_2 + la3*tg53213_2
42 eta2b =~ 1*tg53211_4 + la2*tg53212_4 + la3*tg53213_4
43 eta3b =~ 1*tg53211_6 + la2*tg53212_6 + la3*tg53213_6
44 eta4b =~ 1*tg53211_8 + la2*tg53212_8 + la3*tg53213_8
45
46 # Inclusion of two method factors to deal with correlated uniqueness across time
    points
47 mf1b =~ 1*tg53212_2 + 1*tg53212_4 + 1*tg53212_6 + 1*tg53212_8
48 mf2b =~ 1*tg53213_2 + 1*tg53213_4 + 1*tg53213_6 + 1*tg53213_8
49
50 ## On the between-level, means of the method factors are also included
51 mf1b ~ 1; mf2b ~ 1
52
53 # Variances of the observed variables (measuring eta) are fixed to (almost) zero
54 ## These variables show little variance on the between-level
55 ## If this variance is decomposed into Var(eta) and Var(epsilon),
56 ## the variance of the measurement error is often estimated to be negative
57 ## To deal with this estimation problem, Var(epsilon) is fixed.
58 tg53211_2 ~ 0.0001*tg53211_2
59 tg53212_2 ~ 0.0001*tg53212_2
60 tg53213_2 ~ 0.0001*tg53213_2
61
62 tg53211_4 ~ 0.0001*tg53211_4
63 tg53212_4 ~ 0.0001*tg53212_4
64 tg53213_4 ~ 0.0001*tg53213_4
65
66 tg53211_6 ~ 0.0001*tg53211_6
67 tg53212_6 ~ 0.0001*tg53212_6
68 tg53213_6 ~ 0.0001*tg53213_6
69
70 tg53211_8 ~ 0.0001*tg53211_8
71 tg53212_8 ~ 0.0001*tg53212_8
72 tg53213_8 ~ 0.0001*tg53213_8
73
74 # Means of observed variables (measuring eta) fixed to zero
75 # in order to examine the means of the latent variables
76 tg53211_2 ~ 0*1
77 tg53212_2 ~ 0*1
78 tg53213_2 ~ 0*1

```

```

79  tg53211_4 ~ 0*1
80  tg53212_4 ~ 0*1
81  tg53213_4 ~ 0*1
82  tg53211_6 ~ 0*1
83  tg53212_6 ~ 0*1
84  tg53213_6 ~ 0*1
85  tg53211_8 ~ 0*1
86  tg53212_8 ~ 0*1
87  tg53213_8 ~ 0*1
88
89  # Means of latent state variables eta fixed to zero
90  # in order to examine the means of the growth components
91  eta1b ~ 0*1
92  eta2b ~ 0*1
93  eta3b ~ 0*1
94  eta4b ~ 0*1
95
96  # Variances of the Latent State Variables are Fixed to Zero
97  # Because the Variables are Completely Decomposed into the Growth Components
98  eta1b =~ 0*eta1b
99  eta2b =~ 0*eta2b
100 eta3b =~ 0*eta3b
101 eta4b =~ 0*eta4b
102
103
104 # Growth Components Defined via the Latent State Variables
105 # Using the Inverse Contrast Matrix C
106 # For More Details How to get from C to these coefficients see Appendix B
107 pi1b =~ (1)*eta1b + (1)*eta2b + (1)*eta3b + (1)*eta4b
108 pi2b =~ (0)*eta1b + (.33333)*eta2b + (.33333)*eta3b + (.33333)*eta4b
109 pi3b =~ (0)*eta1b + (-.33333)*eta2b + (.16667)*eta3b + (.16667)*eta4b
110 pi4b =~ (0)*eta1b + (0)*eta2b + (-.5)*eta3b + (.5)*eta4b
111
112 # Include the means of the growth components
113 pi1b ~ 1; pi2b ~ 1; pi3b ~ 1; pi4b ~ 1
114
115 # Regressions of Growth Components on the Predictors
116 # t245403_2 is the perceived burden of examination
117 pi1b ~ t245403_2
118 pi2b ~ t245403_2
119 pi3b ~ t245403_2
120 pi4b ~ t245403_2

```

D. *Mplus* input

The model command in the *Mplus* (L. K. Muthén & Muthén, 1998-2012) input for the complete model including predictors is:

```

1  MODEL:
2  %Within%
3
4  eta1w by tg53211_2@1
5  tg53212_2 (1a2)
6  tg53213_2 (1a3);
7
8  eta2w by tg53211_4@1
9  tg53212_4 (1a2)
10 tg53213_4 (1a3);
11
12 eta3w by tg53211_6@1
13 tg53212_6 (1a2)
14 tg53213_6 (1a3);
15
16 eta4w by tg53211_8@1
17 tg53212_8 (1a2)
18 tg53213_8 (1a3);
19
20 mf1w by tg53212_2@1
21 tg53212_4@1
22 tg53212_6@1
23 tg53212_8@1;
24
25 mf2w by tg53213_2@1
26 tg53213_4@1
27 tg53213_6@1
28 tg53213_8@1;
29
30 eta1w@0 eta2w@0 eta3w@0 eta4w@0;
31
32 pi1w by eta1w@1 eta2w@1 eta3w@1 eta4w@1;
33 pi2w by eta2w@0.33333 eta3w@0.33333 eta4w@0.33333;
34 pi3w by eta2w@-0.33333 eta3w@0.16667 eta4w@0.16667;
35 pi4w by eta3w@-0.5 eta4w@0.5;
36
37 pi1w on t245403_2
38 tg51300_4;
39 pi2w on t245403_2
40 tg51300_4;
41 pi3w on t245403_2
42 tg51300_4;
```

43 pi4w **on** t245403_2
44 tg51300_4;
45
46 %Between%
47
48 eta1b **by** tg53211_2@1
49 tg53212_2 (1a2)
50 tg53213_2 (1a3);
51
52 eta2b **by** tg53211_4@1
53 tg53212_4 (1a2)
54 tg53213_4 (1a3);
55
56 eta3b **by** tg53211_6@1
57 tg53212_6 (1a2)
58 tg53213_6 (1a3);
59
60 eta4b **by** tg53211_8@1
61 tg53212_8 (1a2)
62 tg53213_8 (1a3);
63
64 mf1b **by** tg53212_2@1
65 tg53212_4@1
66 tg53212_6@1
67 tg53212_8@1;
68
69 mf2b **by** tg53213_2@1
70 tg53213_4@1
71 tg53213_6@1
72 tg53213_8@1;
73
74 [mf1b* mf2b*]
75
76 tg53211_2@0;
77 tg53212_2@0;
78 tg53213_2@0;
79
80 tg53211_4@0;
81 tg53212_4@0;
82 tg53213_4@0;
83
84 tg53211_6@0;
85 tg53212_6@0;
86 tg53213_6@0;
87
88 tg53211_8@0;

```

89  tg53212_8@0;
90  tg53213_8@0;
91
92  [tg53211_2@0
93  tg53212_2@0
94  tg53213_2@0];
95
96  [tg53211_4@0
97  tg53212_4@0
98  tg53213_4@0];
99
100 [tg53211_6@0
101 tg53212_6@0
102 tg53213_6@0];
103
104 [tg53211_8@0
105 tg53212_8@0
106 tg53213_8@0];
107
108 [eta1b@0];
109 [eta2b@0];
110 [eta3b@0];
111 [eta4b@0];
112
113 eta1b@0 eta2b@0 eta3b@0 eta4b@0;
114
115 pi1b by eta1b@1 eta2b@1 eta3b@1 eta4b@1;
116 pi2b by eta2b@0.33333 eta3b@0.33333 eta4b@0.33333;
117 pi3b by eta2b@-0.33333 eta3b@0.16667 eta4b@0.16667;
118 pi4b by eta3b@-0.5 eta4b@0.5;
119
120 [pi1b* pi2b* pi3b* pi4b*];
121
122 pi1b on t245403_2;
123 pi2b on t245403_2;
124 pi3b on t245403_2;
125 pi4b on t245403_2;

```
