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Evaluating mathematical creativity: The interplay between multiplicity and insight[1]

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Abstract

This paper presents an original model for evaluation of mathematical creativity. I describe different stages of the model's development and justify critical decisions taken throughout, based on the analysis of the model's implementation. The model incorporates an integrative theoretical framework that was developed based on works devoted to both general and mathematical creativity. The scoring scheme for the evaluation of creativity, which is an important part of the model, combines an examination of both divergent and convergent thinking as reflected in problem solving processes and outcomes. The theoretical connection between creativity and divergent thinking is reflected in the multiplicity component of the model, which is based on the explicit requirement to solve mathematical problems in multiple ways. It is evaluated for fluency and flexibility. The connection between creativity and convergent thinking is reflected in the component of insight, which is based on the possibility to produce insight-based solutions to mathematical problems. I provide examples of the study in which the model is used to examine differences in creativity of students with different levels of excellence in mathematics and different levels of general giftedness.

Key words: Creativity, multiplicity, insight, giftedness, excellence

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[1] This paper is an advanced version of Leikin (2009)

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Background

Creativity in mathematics and mathematics education

The importance of creativity is difficult to overestimate. In a vastly changing world, in which technological and scientific advancements change social networks and individuals' lives, creativity is needed both for adapting to this changing world and for continuing these advancements. Mathematical creativity is a specific type of creativity whose importance is obvious. On the one hand, advances in different branches of mathematics, which research mathematicians bring to life, reflect human intellect. On the other hand, mathematics is one of the central scientific areas that allow sustaining social technological and scientific progress in a variety of areas through offering scientists and Hi-tech specialists a powerful apparatus and models for the analysis of situations, prognoses and processes. School mathematics should provide each and every student with opportunities to get a taste of mathematical creativity and realize his/her creative potential in mathematics.

Haylock (1987) called for greater attention to be paid to creativity in the mathematics classroom. When reviewing the educational literature from 1966 to 1985, Haylock (1987) demonstrated that the subject of creativity is neglected in mathematics education research. Two decades later I reviewed publications from 1999 to 2009 in leading research journals in mathematics education and in gifted education (Leikin, 2009a). This review demonstrated that very few publications were devoted to mathematical creativity.

Fortunately, the mathematics education community has been devoting more attention lately to this issue (see examples in Leikin et al., 2009; Sriraman et al., 2009; Leikin & Pitta-Pantazi, 2013). Mathematics educators have established a new ICME-affiliated International Group for Mathematical Creativity and Giftedness (http://igmcg.org); the ICME and ERME conferences have been devoting the efforts of working groups to this topic with the purpose of raising the mathematics education community's awareness of the fields of mathematical creativity, mathematical potential and mathematical giftedness. Nevertheless, only a small number of empirical studies on creativity associated with mathematics have been carried out.

Guilford (1967) considered the creative process to be based on the combination of convergent thinking, which involves aiming for a single, correct solution to a problem, and divergent thinking, which involves generation of multiple answers to a problem or phenomenon. Torrance (1974) suggested an operative definition of creativity based on four related components: fluency, flexibility, novelty, and elaboration. *Fluency* refers to the continuity of ideas, flow of associations, and use of basic and universal knowledge. *Flexibility* is associated with changing ideas, approaching a problem in various ways, and producing a variety of solutions. *Originality* is characterized by a unique way of thinking and unique products of mental or artistic activity. *Elaboration* refers to the ability to describe, illuminate, and generalize ideas. As creativity is usually viewed as a process that leads to generation of original ideas, the originality component is commonly acknowledged as the main component of creativity.

Another view on the combination of convergent and divergent thinking in creative thinking is found in Cropley (2006), who claims that creative thinking involves two main components: "generation of novelty (via divergent thinking) and evaluation of the novelty (via convergent thinking)" (p. 391). Knowledge is of great importance in the creative process. From the point of view of divergent thinking that includes finding different solutions and interpretations, applying different techniques, and thinking originally and unusually, creativity is one of the learning outcomes (Leikin, 2011). At the same time, for convergent thinking knowledge is of particular importance as a source of ideas, pathways to solutions, and criteria of effectiveness and novelty (Cropley, 2006).

Mathematical creativity is a *specific creativity* that takes into account the logical deductive nature of the field (e.g., Piirto, 1999). As in the case of general creativity, providing a precise and broadly accepted definition of mathematical creativity is extremely difficult and probably impossible to achieve (Haylock, 1987; Leikin, 2009b, Leikin & Pitta-Pantazi, 2013; Mann, 2006). Mann (2006) maintained that analysis of the research attempting to define mathematical creativity demonstrates that the absence of an accepted definition for mathematical creativity hinders research efforts.

Mathematical understanding and insight form the basis of mathematical creation (Ervynck, 1991). Mathematical creativity is one of the characteristics of advanced mathematical thinking, which is reflected in the ability to formulate mathematical objectives and find inherent relationships among them (Ervynck, 1991). Creative products, therefore, lead to the understanding of mathematical relationships and uncover hidden relationships. Not less important is the relationship between mathematical creativity and the beauty in mathematics (Dreyfus & Eisenberg, 1986; Mann, 2006). The mathematical mind seeks elegant products and processes that usually are extremely original and related to insight and, thus, the elegance and beauty of a solution is an indication of mathematical creativity.

Naturally, creativity in school mathematics differs from that of professional mathematicians (Leikin, 2009, Leikin & Lev, 2013). Mathematical creativity in high school students is evaluated with reference to their previous experiences and to the performance of other students who have a similar educational history. I suggest that considering personal creativity as a dynamic characteristic (both personal and social) requires a distinction between *relative* and *absolute* creativity (Leikin, 2009). *Absolute creativity* is associated with discoveries that advance mathematics as a science. *Relative creativity* refers to discoveries by a specific person within a specific reference group. Obviously, school students can offer ideas which are novel with respect to the mathematics they have already learned and to the problems they have solved.

Mathematical creativity and problem solving

The dynamic perspective on mathematical creativity emphasizes the importance of tools that allow the evaluation and development of creativity. The importance of these tools follows from the argument that "the significance of creativity in school mathematics may be minimized because it is not formally assessed on standardized tests, which purport to thoroughly measure mathematical learning" (Chamberlin & Moon, 2005, p. 42). Studies by Chiu (2009) and Kwon, Park, and Park (2006) suggested ways to evaluate mathematical creativity. Some limitations of the evaluation tools presented in these studies may be seen in the close connection between the evaluation tools and the type of activities the researchers considered to be creative. Consequently, these studies demonstrate that the recommended instructional activities are teachable. Livne, Livne, and Milgram (1998) proposed a universal questionnaire for measuring mathematical creativity that did not address the relative characteristics of creativity or its multidimensional structure.

Following Torrance (1974), Silver (1997) suggested developing creativity through problem solving as follows: *Fluency* is developed by generating multiple ideas, multiple answers to a problem (when such exist), exploring situations, and raising multiple ideas; *flexibility* is advanced by generating new solutions when at least one has already been produced; and *novelty* is advanced by exploring many solutions to a problem and generating a new one.

Ervynck (1991), who considered creativity to be a critical component of advanced mathematical thinking, stressed that mathematical creativity is based on the previous experiences of an individual and requires "making a step forward in a new direction" (p. 42). He suggested that on the way toward creative activity there are at least two necessary stages: a preliminary, technical stage and an algorithmic activity stage. Ervynck identified three different levels of creativity. Level 1 contains an algorithmic solution to a problem; Level 2 involves modeling a situation and may include solving a word problem with a graph or a linear diagram; Level 3 employs sophisticated methods usually based on assumptions embedded in the problem, and makes use of the problem's internal structure and insight. Since categorisation of types of solutions according the levels of creativity suggested by Ervynck (1991) is based on the connection between the solutions and solver's previous mathematical experiences, this categorisation fits the definition of relative creativity, in general, and of originality, in particular.

The current paper represents different steps in the design of a multidimensional model for the evaluation of mathematical creativity that takes into account the relative nature of creativity. It draws on the views of Ervynck (1991), Krutetskii (1976), Polya (1973), and Silver (1997) who claim that solving mathematical problems in multiple ways is closely related to personal mathematical creativity, and suggest evaluating mathematical creativity by means of multiple-solution tasks (MSTs). The model contains operational definitions and a corresponding scoring scheme to evaluate creativity based on three dimensions (originality, fluency, and flexibility), as suggested by Torrance (1974). To evaluate originality it uses Ervynck's insight-related levels of creativity in combination with the conventionality of the solutions, which includes the students' history of mathematical education.

MSTs and solution spaces

A multiple-solution task (MST) is an assignment in which a student is explicitly required to solve a mathematical problem in different ways. Solutions to the same problem are considered to be different if they are based on: (a) different representations of some mathematical concepts involved in the task, (b) different properties (definitions or theorems) of mathematical objects within a particular field, or (c) different properties of a mathematical object in different fields (see the definition and various examples of MSTs in Leikin, 2006, 2009) (see example in Figure 1).

In Leikin (2007) I suggest the notion of *solution spaces*, which enables researchers to examine the various aspects of problem-solving performance using MSTs. *Expert solution spaces* include the most complete set of solutions to a problem known at a particular time. They can also be conceived as a set of solutions that expert mathematicians can suggest to the problem. In school mathematics, expert solution spaces include *conventional solution space*s, which are those generally recommended by the curriculum, as displayed in textbooks, and usually taught by the teachers. By contrast, *unconventional solution spaces* include solutions based on strategies usually not prescribed by the school curriculum, or which the curriculum recommends with respect to a different type of problem. *Individual solution spaces* are collections of solutions produced by an individual to a particular problem. With respect to the ability of a person to find solutions independently, we distinguish between *available individual solution spaces*, which include solutions that individuals can present on the spot or with some effort without help from others, and *potential solution spaces*, which include solutions that solvers produce with help from others. Solutions derived from the potential solution spaces correspond to the personal ZPD (Vygotsky, 1978). Finally, *collective solution spaces* are a combination of the solutions produced by a group of individuals. Collective solution spaces are usually broader than individual solution spaces within a particular community, and form one of the main sources for the development of individual solution spaces. Both individual and collective solution spaces are subsets of expert solution spaces.

Solution spaces are used here as a tool for exploring the students' mathematical creativity and for the assessment of the potential of a task to evaluate mathematical creativity.

Task 1: Solve the system in as many ways as possible: $\begin{cases} 3x+2y=10 \\ 2x+3y=10 \end{cases}$

Solutions

1. Algebraic solutions:
    1.1. Linear combination
    1.2. Substitution for x (y)
    1.3. Equalizing algebraic expressions for x (y)
2. Graphing
3. Matrices
4. Symmetry considerations

Figure 1:
MST example.

Modeling mathematical creativity with MST

Runco and Acar (2012) stressed the significance of developing scoring systems for divergent thinking tasks. In this section, I present in detail the development of the scoring scheme for the evaluation of creativity with MSTs. This scoring scheme also addresses convergent thinking as a key component of creativity by including insight as an indicator of convergent thinking.

Initial 2-4-6 scoring scheme

Based on the theoretical assumption and definitions that connect mathematical creativity with solving mathematical problems in multiple ways, Leikin and Lev examined differences in the creativity of gifted and non-gifted students who excel in mathematics (Leikin & Lev, 2007). We developed a tool that contains a set of mathematical problems and a scoring scheme for evaluating the students' problem-solving performance on MSTs. We examined *novelty* of solutions according to their conventionality (see conventional and non-conventional solution spaces above), their availability, and repetition. The originality of the students' individual solutions (produced without hints) was scored 2, 4, and 6 according to the level of their conventionality, whereas the same solutions produced with hints received scores of 1, 2, and 3.

Flexibility was evaluated by the number of non-repeating solutions in the available and potential individual solution spaces. *Fluency* was evaluated with respect to the time spent by the students to produce the solutions. Using this model and the 2-4-6 scoring scheme we demonstrated that the creativity of gifted students was higher than that of regular students on every type of task, and that their creativity differed from that of their expert non-gifted counterparts on non-routine tasks only. Similar scoring schemes for the evaluation of mathematical creativity with MSTs are used in Kontoyianni, Kattou, Pitta-Pantazi, and Christou (2013).

Limitations of 2-4-6 scoring scheme

Deeper analysis of the results of the study by Leikin & Lev (2007) revealed the following limitations of the suggested scoring scheme:

When evaluating creativity using the originality, fluency, and flexibility components with a 2-4-6 scoring scheme, the score given for an individual solution space of a problem reflected the problem solving product, which did not allow for reproducing the flexibility and originality of the problem-solving process. Thus, the objective was to develop a scoring scheme that would allow analysing both the problem-solving process and the problem-solving product based on the final score in flexibility and originality.

Refined model

A refined model (Leikin, 2009) included a new scoring scheme and more precise definitions of the creativity components. This model not only made it possible to evaluate the students' personal mathematical creativity but also to estimate the efficiency of MSTs in evaluating creativity. The final score that students received reflected the flexibility and originality integrated in the problem-solving process as well as in the problem-solving product. The model allowed evaluation of creativity for individuals and groups of individuals, as well as for the tasks themselves. The creativity embedded in a task is evaluated based on its expert solution spaces. The model was implemented in a number of studies in which students' creativity was evaluated in different research settings (Guberman & Leikin, 2013; Levav-Waynberg & Leikin, 2009, 2012; Leikin & Kloss, 2011; Leikin, Levav-Waynberg, & Guberman, 2011; Leikin & Lev 2013; Lev & Leikin, 2013). Through performance of these studies some additional modifications of Leikin's (2009) model were performed. In what follows I describe the most updated version of the model, addressing these latest adjustments.

Fluency

Fluency (N) is usually measured by the number of appropriate ways produced for solving a problem insofar as it reflects the pace at which solving proceeds and the switches taking place between different ways of solutions. A student's fluency on a written test is detected by the number of appropriate solutions in his/her individual solution space. The notion of appropriateness has replaced the notion of correctness (e.g., Leikin, 2009 vs. Levav-Waynberg & Leikin, 2012) to allow evaluation of reasonable ways of solving a problem that potentially lead to the correct solution outcome regardless of the minor mistakes made by a solver. The fluency embedded in an MST is the number of solutions in the expert solution space.

Flexibility

When evaluating flexibility (Flx) we refer to different groups of ways of solving (simple solutions) an MST. Two solutions belong to separate groups if they employ solution strategies based on different representations, properties (theorems, definitions, or auxiliary constructions), or branches of mathematics. The flexibility embedded in an MST is evaluated based on the groups of solutions in the expert solution space. Flexibility of a student's performance on an MST is evaluated based on the solution strategies in the individual solution space. We suggest using a decimal basis for evaluation of flexibility as follows: $Flx_1 = 10$ for the first appropriate solution. For each consecutive solution there are several scores: $Flx_i = 10$ if a solution belongs to a group of solutions different from ones to which the solution(s) performed previously belong (for example, Solution 2 produced after Solution 1, Figure 1). $Flx_i = 1$ if the solution belongs to one of the previously used groups but has a clear minor distinction (for example, Solution 1.3 produced after Solution 1.1, Figure 1). $Flx_i = 0.1$ if the solution is almost identical with (one of the) previously performed solutions (for example, Solution 1.2 performed twice – for x and for y). A score of 0.1, which is a negative (-1) power of 10, reflects the lack of students' critical reasoning, which is essential for mental flexibility, and the inability to recognize the two produced solutions as being identical. A student's *total flexibility score* on a problem is the sum of his/her flexibility on the solutions in the student's individual solution space - $Flx = \sum_{i=1}^{N} Flx_i$ where N is fluency score.

The *decimal basis* we use for scoring flexibility reflects both the problem solving product and the process. For example, if the total flexibility score for a solution space is 31.2, we know that it includes 3 solutions that belong to different solution groups, 1 solution that uses a solution strategy from one of the former groups exhibiting a minor but essential difference, and 2 solutions that repeat previous ones.

Originality

When evaluating originality we combine "relative" evaluation of originality with "absolute" evaluation that refers to insight embedded in the solution strategy produced by the student. Relative evaluation of originality is performed with respect to the conventionality of a solution in a particular group of students with a similar educational history. For this purpose we compare individual solution spaces with the collective solution space of the reference group through calculation of the percentage (*P*) of the students in the group that produces a particular solution. Absolute evaluation, which is based on the level of insight involved in the solution process (c.f., Ervynck, 1991), prevents evaluation of an algorithmic solution (which is obviously a learned one) performed by a student as an original one, even if only one student in his/her reference group produced the solution. The insight-related originality reflects, to a great extent, the convergent reasoning of the individuals.

As in the case of flexibility, we used a decimal basis for evaluation of originality as follows: Originality of a particular solution is scored with $Or_i = 10$ for an insight-based unconventional solution (e.g., Solution 4, Figure 1). Usually solutions of this type are produced by not more than 15% of students in a particular reference group. A score of $Or_i = 1$ is given for a model-based solution or a solution which implies a solution strategy learned in a different context (e.g., Solution 2, Figure 1). Relative evaluation of such kinds of solutions belongs to the domain of $15\% \le P < 40\%$, where P is the percentage of students in the reference group who produced this kind of solution (e.g., Solution 1, Figure 1). Algorithm-based or conventional (i.e., definitely learned) solutions are scored with $Or_i = 0.1$. Solutions of this kind are usually produced by over 40% of students in the reference group. A student's *total originality score* on a problem is calculated as the sum of the student's originality on the solutions in the student's individual solution space. The total originality embedded in a task is the sum of originality scores of all the solutions in the expert solution space, $Or = \sum_{i=1}^{n} Or_i$ where n is the number of appropriate solutions in the corresponding space.

In the *decimal basis* we used in scoring, the total score indicates the originality of the solutions in the focal solution spaces. For example, a total originality score of 21.3 means that the evaluated solutions space includes 2 insight-based/non-conventional solutions, 1 solution that is partly unconventional, and 3 algorithm-based solutions. The decision regarding 15% and 40% as borderlines between the different levels of originality was based on previous experiments. We also compared the results of written tests with the students' performance in individual interviews and classroom discussions. We found that on written tests these percentages (15% and 40%) match quite accurately the various levels of originality of solutions produced and presented both during the interviews and in the classroom discussion.

Note here that a more recent adjustment has been performed for the evaluation of originality. In Leikin and Lev (2013) and Lev and Leikin (2013) we discovered the essentiality of the combination of relative evaluation of originality suggested earlier in Leikin (2009) and used in earlier studies (e.g., Levav-Waynberg & Leikin, 2012) with evaluation of insight embedded in the solution strategy produced by the student. When evaluating originality of solutions in our latest study we observed that in some mid-level classes of students, when a problem was generally solved by less than 15% of students, their originality score produced by relevant evaluation was high (Or = 10) even though this solution was algorithm-based. Thus, we argue that evaluation of originality requires both relative (quantitative) and absolute (qualitative) examination of the solutions.

Creativity

The creativity (Cr) of a particular solution is the product of the solution's originality and flexibility: $Cr_i = Flx_i \times Or_i$. We use the product of the flexibility and originality scores to evaluate creativity based on the following consideration: Suppose that a student (Tom) produces a solution flexibly ($Flx_k = 10$), in other words the solution k is the first solution or belongs to a new group of solutions. If Tom produces an original solution ($Or_k = 10$) flexibly ($Flx_k = 10$), then his creativity on this solution is scored $Cr_k = 100$. A solution (in the same group) that is similar to one of the previously performed solutions cannot be considered as a creative act. Thus, when Tom performs an original solution ($Or_m = 10$) that is similar to one produced earlier, his flexibility is scored $Flx_m = 1$ or $Flx_m = 0.1$. The creativity score is then $Cr_m = 10$ or $Cr_m = 1$, a score that indicates a different level of creativity for the solution process. When a student produces an unoriginal solution ($Or_n = 1$ or $Or_n = 0.1$) flexibly ($Flx_n = 10$), it results in a creativity score that expresses a medium or low level of creativity ($Cr_n = 10$ or $Cr_n = 1$). Repeating unoriginal solutions scores Cr = 0.1 or Cr = 0.01 and indicates that a student does not see the similarity between the solutions and produces only solutions learned in the classroom.

The total *creativity score* on a MST is the sum of the creativity scores on each solution in the individual solution space of a problem: $Cr = \sum_{i=1}^{n} Flx_i \times Or_i$.

The decision to evaluate the creativity of a solution as the product of flexibility and originality scores and to consider the total score as the sum of the scores on different solutions helped us in the decision to evaluate the flexibility of the first solution to a given problem as $Flx_1 = 10$ (see also Leikin, 2009). We assumed that the creativity of two individual solution spaces that contain identical sets of solutions should be scored equal-

	Fluency	Flexibility	Originality	Creativity
Scores per solution	1	• $Flx_1 = 10$ for the first solution • $Flx_i = 10$ solutions from a different group of strategies • $Flx_i = 1$ similar strategy but a different representation • $Flx_i = 0.1$ the same strategy, the same representation	• $Or_i = 10$ for insight/ unconventional solution or $P < 15\%$ • $Or_i = 1$ for model-based/ partly unconventional solution or $15\% \le P < 40\%$ • $Or_i = 0.1$ for algorithm-based/ conventional solution or $Or_i = 0.1$ $P \ge 40\%$	$Flx_i \times Or_i$
Total score	n	$Flx = \sum_{i=1}^{n} Flx_i$	$Or = \sum_{i=1}^{n} Or_i$	$\sum_{i=1}^{n} Flx_i \times Or_i$
Final creativity score	$Cr = \left(\sum_{i=1}^{n} Flx_i \times Or_i \right)$			
n is the total number of appropriate solutions $P = (m_j / n) \cdot 100\%$ where m_j *is the number of students who used strategy* j				

Figure 2:
Evaluation of creativity in different contexts.

ly. Suppose we assigned a score of $Flx_1 = 1$. If on a particular problem a student (Tom) produces two solutions that belong to two different groups, his flexibility is scored $Flx_1 = 1, Flx_2 = 10$. Suppose that the originality scores of these two solutions are $Or_1 = 1, Or_2 = 10$. Then Tom's total creativity score is $Cr = 101$. If another student (Harry) performs the same two solutions but in a different order, for him $Flx_1 = 1, Flx_2 = 10$ and $Or_1 = 10, Or_2 = 1$, and his total creativity score is 20. Tom's and Harry's individual solution spaces are identical and, therefore, their creativity should also be scored as equal. This conflict may be solved by scoring the flexibility of the first solution $Flx_1 = 10$. In this case both Tom and Harry receive a total creativity score of 110. Moreover, we liked the idea that if a student produced only one solution but it was an original one, his or her creativity should be scored 100.

Implementation of the Model – an example

The model was implemented in a number of studies in which students' creativity was evaluated in different research settings (Guberman & Leikin, 2013; Levav-Waynberg & Leikin, 2009, 2012; Leikin & Kloss, 2011; Leikin, Levav-Waynberg, & Guberman, 2011). I present here the latest implementation performed in collaboration with Miri Lev (Lev & Leikin, 2013). One of the study goals was to examine relationships between mathematical creativity, general giftedness, and mathematical excellence. Task 1 (Figure 1) was included in the test. In addition to the evaluation of creativity components according to the scoring scheme (Figure 2), we examined correctness of the solutions with 25 points for a complete solution.

A sample of 191 (students subdivided into four experimental groups – see Table 1) was chosen out of a population of 1200 10^{th} and 11^{th} grade students (16-17 years old). The sampling procedure was directed towards investigating the effect of General Giftedness and Excellence in Mathematics (G and EM factors).

G factor: Students for G groups were mainly chosen from classes for gifted students (IQ > 130). Additionally, the entire research population was examined using Raven’s Advanced Progressive Matrix Test (RPMT) (Raven, Raven, & Court, 2000).

EM factor: All 1200 students studied mathematics at high and regular levels (HL, RL). The level of instruction is determined by students’ mathematical achievements in earlier grades. Instruction at HL differs from that at RL in terms of the depth of the learning material and the complexity of the mathematical problem-solving involved. Additionally, excellence in mathematics is examined using the SAT-M (Scholastic Assessment Test in Mathematics, adopted from Koichu, 2003).

Table 1:
Target population.

	Gifted (G) IQ > 130 Raven > 27/30	Non-Gifted (NG) Raven < 26/30	Total
Excelling in Math (EM) SAT-M > 26 or HL in mathematics with math score > 92	G-EM *n* = 38	NG-EM *n* = 51	87
Non-excelling in Math (NEM) SAT-M < 22 and RL in mathematics with math score > 90 or HL in mathematics with math score < 80.	G-NEM *n* = 38	NG-NEM *n* = 57	95
Total	76	108	183

Results

Multivariate analysis of variance tests (MANOVAs) were used to compare the scores on each component of creativity that participants received for each problem. *Between-subject differences* were examined for each one of the problems and each one of the creativity components for G factor, EM factor and interactions between G and EM factors. *Within-subject differences* were examined for performance on the different tasks.

Table 2 presents the percentage of students with different levels of fluency (the number of appropriate solutions produced by a student) and flexibility (the number of solutions from different groups). We learn from these data that students in all the groups were successful, fluent and flexible in solving the system of equation. From Table 2 we learn that though there is connection between fluency and flexibility in students' problem solving performance, they measure different kinds of mental ability. Production of multiple solutions does not mean production of different multiple solutions. Clearly students from the G-EM group differed meaningfully in their flexibility when solving both problems. Participants from the G-EM group differed from participants of all other groups in the fluency and flexibility of their problem solving performance.

Table 3, which presents the Means and SD that we obtained for all the examined criteria on both problems, provides additional support for the observation of the specific qualities of mathematical reasoning in G-EM students. Only G-EM students produced insight-based solutions; this means that only students from this group received a high-level (10) originality score.

MANOVAs demonstrate effects of EM and G factors on all the examined criteria (Table 4). A significant main effect of the G factor was found for all the criteria, while the EM factor has a significant main effect on flexibility only. We also found an interaction between EM and G factors with respect to students' flexibility related to solving the system of equations. G factor strengthens the effect of EM factor; that is G-EM students are significantly more flexible than their NG-EM counterparts, whereas no significant differences appear in flexibility of EM and NEM students among NG students (see also Table 3).

We hypothesize that in the fluency-flexibility-originality triad, fluency and flexibility are of a dynamic nature, whereas originality is a "gift". We demonstrate that originality appears to be the strongest component in determining creativity.

The strength of the relationship between creativity and originality can be considered as validating our model, being consistent with the view of creativity as an invention of new products or procedures. At the same time, our studies demonstrate that this view is true for both absolute and relative creativity. Based on the research findings, we hypothesize that one of the ways of identifying mathematically gifted students is by means of originality of their ideas and solutions. Systematic research should be performed to examine our hypotheses.

Table 2:
Fluency and Flexibility.

No. of solutions (Flu) / No. of groups of solutions (Flx)		0	1	2	3	4	5	6
G-EM (n = 38)	Flu	0	0	7.9	71	21		
	Flx	0	61	26	13			
G-NEM (n = 38)	Flu	5.3	0	5.3	68	16	2.6	2.6
	Flx	5.3	82	13				
NG-EM (n = 51)	Flu	2	2	9.8	78	7.8		
	Flx	2	94	3.9				
NG-NEM (n = 57)	Flu	1.8	14	14	61	7	1.8	
	Flx	1.8	91	7				

Table 3:
Means and SD.

N		G-EM		G-NEM		NG-EM		NG-NEM	
		n = 38		n = 38		n = 51		n = 57	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\begin{cases} 3x+4y=14 \\ 4x+3y=14 \end{cases}$	Cor	25	0	23.68	5.657	24.51	3.501	23.77	4.359
	Flu	3.13	0.529	3.13	1.212	2.88	0.653	2.63	0.938
	Flx	16.42	6.948	12.53	4.759	11.51	2.575	11.73	3.232
	Or	3.18	5.024	1.66	3.391	0.68	1.944	0.989	2.621
	Cr	30.09	50.727	14.54	34.18	5.03	19.602	8.294	25.74

Table 4:
Effects of G and EM factors

Between-Subject Effects			G -factor	EM-factor	G×EM
		F(3,183)	F(1,183)	F(1,183)	F(1,183)
$\begin{cases} 3x+4y=14 \\ 4x+3y=14 \end{cases}$	Cor	1.043	.114	2.955	234.
	Flu	*3.633	8.321**	.932	932.
	Flx	***11.025	18.492***	7.626**	9.513**
	Or	**4.891	10.463**	1.501	3.469
	Cr	**4.830	10.084**	1.554	3.641

Summary

This paper presents a model for evaluation of mathematical creativity. I describe the evolution of the model and justify its structure. The theoretical connection between creativity and divergent thinking is reflected in the multiplicity component of the model, which is based on the explicit requirement to solve mathematical problems in multiple ways. The scoring scheme for evaluation of creativity, which is a part of the model, refers to the fluency, flexibility and originality of students' mathematical thinking reflected in the problem-solving strategies they use. The ability to produce insight-based solutions is an intrinsic requirement of the mathematical problems that can and should be used for the evaluation of creativity using the model. The usefulness of the model is exemplified in the study by Lev and Leikin (2013).

Other studies that were performed using the model also demonstrate its usefulness in evaluating the development of students' creativity in different educational frameworks (Guberman & Leikin, 2013; Levav-Waynberg & Leikin, 2012). These studies demonstrate that both high achievers and mid-achievers in mathematics significantly improve their problem-solving accuracy, fluency and flexibility in an instructional environment that is directed towards development of mathematical creativity. At the same time, the improvement of fluency and flexibility is significantly greater for the high level participants. We also demonstrate that an increase in flexibility is accompanied by a decrease in originality on the group level, while only a small number of participants increase their originality on the individual level (Levav-Waynberg & Leikin, 2012). Following these findings, we question the possibility of developing originality and hypothesize that in the originality-fluency-flexibility triad, fluency and flexibility are of a dynamic nature, whereas originality is of the “gift” type.

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